Approximating the Traffic Grooming Problem*
(Extended Abstract)

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Abstract. The problem of grooming is central in studies of optical networks. In graph-theoretic terms, this can be viewed as assigning colors to the lightpaths so that at most g of them (g being the *grooming factor*) can share one edge. The cost of a coloring is the number of optical switches (ADMs); each lightpath uses two ADM's, one at each endpoint, and in case g lightpaths of the same wavelength enter through the same edge to one node, they can all use the same ADM (thus saving $g-1$ ADMs). The goal is to minimize the total number of ADMs. This problem was shown to be NP-complete for $g=1$ and for a general g. Exact solutions are known for some specific cases, and approximation algorithms for certain topologies exist for $g=1$. We present an approximation algorithm for this problem. For every value of g the running time of the algorithm is polynomial in the input size, and its approximation ratio for a wide variety of network topologies - including the ring topology - is shown to be $2\ln g + o(\ln g)$. This is the first approximation algorithm for the grooming problem with a general grooming factor g.

Keywords: Wavelength Assignment, Wavelength Division Multiplexing(WDM), Optical Networks,Add-Drop Multiplexer(ADM), Traffic Grooming.

1 Introduction

1.1 Background

Optical wavelength-division multiplexing (WDM) is today the most promising technology, that enables us to deal with the enormous growth of traffic in communication networks, like the Internet. A communication between a pair of nodes is done via a *lightpath*, which is assigned a certain wavelength. In graph-theoretic terms, a lightpath is a simple path in the network, with a color assigned to it.

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Most of the studies in optical networks dealt with the issue of assigning colors to lightpaths, so that every two lightpaths that share an edge get different colors.

When the various parameters comprising the switching mechanism in these networks became clearer, the focus of studies shifted, and today a large portion of the studies concentrates with the total hardware cost. The key point here is that each lightpath uses two ADM's, one at each endpoint. If two adjacent lightpaths are assigned the same wavelength, then they can use the same ADM. An ADM may be shared by at most two lightpaths. The total cost considered is the total number of ADMs. Lightpaths sharing ADM's in a common endpoint can be thought as concatenated, so that they form longer paths or cycles. These paths/cycles do not use any edge $e \in E$ twice, for otherwise they cannot use the same wavelength which is a necessary condition to share ADM's.

Moreover, in studying the hardware cost, the issue of *grooming* became central. This problem stems from the fact that the network usually supports traffic that is at rates which are lower than the full wavelength capacity, and therefore the network operator has to be able to put together (= groom) low-capacity demands into the high capacity fibers. In graph-theoretic terms, this can viewed as assigning colors to the lightpaths so that at most g of them (g being the *grooming factor*) can share one edge. In terms of ADMs, each lightpath uses two ADM's, one at each endpoint, and in case g lightpaths of the same wavelength enter through the same edge to one node, they can all use the same ADM (thus saving $g - 1$ ADMs). The goal is to minimize the total number of ADMs. Note that the above coloring problem is simply the case of $g = 1$.

We note that we deal with the *single hop* problem, where a connection is carried along one wavelength. A nice review on traffic grooming problems can be found in [1].

1.2 Previous Work

The problem of minimizing the number of ADMs for the case $g = 1$ was introduced in [2] for ring topology. The problem was shown to be NP-complete for ring networks in [3]. An approximation algorithm for the ring topology with approximation ratio of $3/2$ was presented in [4], and was improved in [5, 6] to $10/7 + \epsilon$ and $10/7$ respectively. For a general topology [3] describes an algorithm with approximation ratio of $8/5$. The same problem was studied in [7], and an algorithm with approximation ratio $3/2 + \epsilon$ was presented.

The notion of traffic grooming ($g > 1$) was introduced in [8] for the ring topology. The problem was shown to be NP-complete in [9] for ring networks and a general g. The uniform all-to-all traffic case, in which there is the same demand between each pair of nodes, is studied in [9, 10] for various values of g; an optimal construction for the uniform all-to-all problem, for the case $g = 2$ in a path network was given in [11].

The hardness results of [3, 9] are for $g = 1$ and for general g, respectively. NP-completeness results for ring and path networks are shown in [12] for general values of g (in the strong sense) and for any fixed value of g.

1.3 Our Contribution

We present an approximation algorithm for the general instance of the traffic grooming problem, namely general topology and general set of requests. The approximation ratio of our algorithm is $2\ln g + o(\ln g)$ in ring networks, with arbitrary set of requests. The ring topology is the most widely studied topology due to its implementation in SONET networks. Therefore and for matter of presentation, our discussion deals only with ring topologies. The extensions are briefly discussed in Section 5. Note that the approximation ratio of any algorithm for this problem is between 1 and $2g$. To the best of our knowledge this is the first approximation algorithm for the grooming problem with a general grooming factor g. In Section 2 we describe the problem and make some preliminary observations. The algorithm presented in Section 3, and analyzed in Section 4. We conclude in Section 5 with possible extensions of this result and some open problems. Some proofs are sketched or omitted in this Extended Abstract.

2 Problem Definition and Basic Observations

An instance of the *traffic grooming problem* is a triple (G, P, g) where $G = (V, E)$ is an undirected graph, P is a set of simple paths in G and g is a positive integer, namely the grooming factor.

Given such an instance we define the following:

Definition 1. *Given a subset $Q \subseteq P$ and an edge $e \in E$, Q_e is the set of paths from Q using edge e. $l_Q(e)$ is the number of these paths, or in networking terminology, the load induced on the edge e by the paths in Q. L_Q is the maximum load induced by the paths in Q on any edge of G. When $Q = P$, we will omit the indices and simply write $l(e)$ and L instead of $l_P(e)$ and L_P respectively. Formally,*

$$\forall Q \subseteq P, \forall e \in E :$$
$$Q_e \stackrel{def}{=} \{p \in Q | e \in p\}$$
$$l_Q(e) \stackrel{def}{=} |Q_e|$$
$$L_Q \stackrel{def}{=} \max_{e \in E} l_Q(e)$$

Definition 2. *A coloring (or wavelength assignment) of (G, P) is a function $w : P \mapsto \mathbb{N}^+ = \{1, 2, ...\}$. We extend the definition of w on any subset Q of P as $w(Q) = \cup_{p \in Q} w(p)$. For a coloring w, a color λ and any $Q \subseteq P$, Q^w_λ is the subset of paths from Q colored λ by w and $Q^w_{e,\lambda}$ is the set of paths from Q, using edge e and colored λ by w. Formally,*

$$Q^w_\lambda \stackrel{def}{=} w^{-1}(\lambda) \cap Q = \{p \in Q | w(p) = \lambda\}$$
$$Q^w_{e,\lambda} \stackrel{def}{=} Q_e \cap Q^w_\lambda.$$

Definition 3. *A proper coloring (or wavelength assignment) w of (G,P,g) is a coloring of P, in which for any edge e at most g paths using e are colored with the same color. Formally, $\forall \lambda \in \mathbb{N}^+, L_{P^w_\lambda} \leq g$.*

Definition 4. *A coloring w is a W-coloring of $Q \subseteq P$, if it colors the paths of Q using exactly W colors. Formally, if $|w(Q)| = W$. A set Q is W-colorable if there exists a proper W-coloring for it.*

For a W-coloring of P, we will assume w.l.o.g. that $w(P) = 1, 2, ..., W$.

Observe that a set $Q \subseteq P$ is 1-colorable iff $L_Q \leq g$.

Now we define the cost function $\#ADM$, under the assumption that G is a cycle.

Definition 5. *For a coloring w of P, a subset $Q \subseteq P$ and a node $v \in V$, Q_v is the subset of paths from Q having an endpoint in v. $Q^w_{v,\lambda}$ is the subset of paths from Q_v colored λ by w. $\#ADM^w_\lambda(v)$ is the number of ADM's operating at wavelength λ at node v.*

For each pair $v \in V, \lambda \in \{1, 2, ..., W\}$ we need one ADM operating at wavelength λ in node v iff there is at least one path colored λ among the paths having an endpoint at v. Formally,

$$
\begin{aligned}
Q_v &\stackrel{def}{=} \{p \in Q | v \textit{ is an endpoint of } p\} \\
touches(Q, v) &\stackrel{def}{=} \begin{cases} 0 \textit{ if } Q_v = \emptyset \\ 1 \textit{ otherwise} \end{cases} \\
endpoints(Q) &\stackrel{def}{=} \sum_{v \in V} touches(Q, v) \\
Q^w_{v,\lambda} &\stackrel{def}{=} Q_v \cap Q^w_\lambda \\
\#ADM^w_\lambda(v) &\stackrel{def}{=} touches(P^w_\lambda, v) \\
\#ADM^w_\lambda(Q) &\stackrel{def}{=} endpoints(Q^w_\lambda) \\
\#ADM^w_\lambda &\stackrel{def}{=} \#ADM^w_\lambda(P) \\
\#ADM^w &\stackrel{def}{=} \sum_\lambda \#ADM^w_\lambda
\end{aligned}
$$

Definition 6. *For any subset $Q \subseteq P$ and any subset $U \subseteq V$, Q_U is the set of paths in Q having at least one endpoint in U. Formally,*

$$Q_U \stackrel{def}{=} \bigcup_{u \in U} Q_u.$$

The traffic grooming problem is the optimization problem of finding a proper coloring w of (G, P, g) minimizing $\#ADM^w$.

Observe that *endpoints* and consequently $\#ADM^w_\lambda$ are monotone non decreasing functions. Formally, if $R \subseteq Q \subseteq P$ then

$$
\begin{aligned}
endpoints(R) &\leq endpoints(Q) \\
\#ADM^w_\lambda(R) &\leq \#ADM^w_\lambda(Q).
\end{aligned}
$$

3 Algorithm GROOMBYSC(k)

Given an instance (G, P, g) of the traffic grooming problem, our algorithm has a parameter k which depends only on g. The value of k will be determined in the analysis (see Section 4).

The algorithm has three phases. During phase 1 it computes 1-colorable sets and their corresponding weights. It considers subsets of the paths P, of size at most $k \cdot g$. Whenever a 1-colorable set is found, it is added to the list of relevant sets, together with its corresponding weight. In phase 2 it finds a set cover of P using subsets calculated in phase 1. It uses the GREEDYSC approximation algorithm for the minimum weight set cover problem presented in [13]. In phase 3 it transforms the set cover into a partition by eliminating intersections, then colors the paths according the partition. Each set in the partition is colored with one color.

1. Phase 1- Prepare the input for GREEDY:

    $S \leftarrow \emptyset$
    For each $U \subseteq V$, such that $|U| \leq k$ {
        For each $Q \subseteq P_U$, such that $|Q| \leq k \cdot g$ {
            If Q is 1-colorable then {
                $S \leftarrow S \cup \{Q\}$
                $weight[Q] = endpoints(Q)$ // weight[] is an associative
                // array containing a weight for each set
            }
        }
    }

2. Phase 2- Run GREEDYSC:

    $SC \leftarrow GREEDYSC(S, weight)$.// Assume w.l.o.g $SC = \{S_1, S_2, ..., S_W\}$

3. Phase 3- Transform the Set Cover SC into a Partition $PART$:

    $PART \leftarrow \emptyset$
    For $i = 1$ to W $\{PART_i \leftarrow S_i\}$
    As long as there are two intersecting sets $PART_i, PART_j$ {
        $PART_i \leftarrow PART_i \setminus PART_j$
    }
    For $\lambda = 1$ to W {
        $PART \leftarrow PART \cup \{PART_\lambda\}$
        For each $p \in PART_\lambda \{w(p) = \lambda\}$
    }

4 Analysis

4.1 Correctness

Claim. w calculated by the algorithm is a coloring.

Proof. During phase 1, each path $p \in P$ is included at least in one set $Q \in S$. This is because the set $\{p\}$ is considered during the loop and it is clearly found to be 1-colorable. As SC is calculated in phase 2 a set cover of these sets, p is an element of at least one set $S_i \in SC$. During phase 3 intersections are eliminated, therefore p is an element of exactly one set of $PART$. Therefore each p is assigned exactly one value $w(p)$ during phase 3. □

Lemma 1. *w calculated by the algorithm is a proper coloring.*

Proof. For every color $\lambda \in \{1, 2, ..., W\}$ the set of paths colored λ is exactly $PART_\lambda$. It suffices to show that the sets $PART_\lambda$ are 1-colorable.

A subset of an x-colorable set is x-colorable. By the code of phase 3 $PART_\lambda \subseteq S_\lambda$. By phase 1, S_λ is 1-colorable, therefore $PART_\lambda$ is 1-colorable. □

4.2 Running Time

Claim. The running time of $GROOMBYSC(k)$ is polynomial in $n = |P|$ and $m = |E|$, for any given g and for all instances (G, P, g).

Proof. We will show that the running time of each one of the three phases is $poly(n, m)$.

- **Phase 1:**
  The number of subsets of P considered during the first phase is $O(n^{g \cdot k})$ since their sizes are at most $g \cdot k$. To check whether a set is 1-colorable takes $O(g \cdot k \cdot m)$ time. To calculate $endpoints(Q)$ can be done in $O(g \cdot k \log |Q|) = O(g \cdot k \cdot \log m)$ time.
  For any constant g, k is determined as a function of g only. Then $g \cdot k$ is a constant. Therefore the running time of phase 1 is polynomial in n and m for any given g.
- **Phase 2:**
  The number of the sets in S is at most $n^{g \cdot k}$. The running time of GREEDYSC is polynomial in $|S|$ and $|P|$, namely $poly(n^{g \cdot k}, n) = poly(n)$.
- **Phase 3:**
  The running time of phase 3 is polynomial in the size of the cover which is in turn polynomial in n. □

4.3 Approximation Ratio

Lemma 2. *Let $H_n = 1 + \frac{1}{2} + ... + \frac{1}{n}$ be the n-th harmonic number. $GROOMBYSC(k)$ is a $H_{g \cdot k}(1 + \frac{2g}{k})$ approximation algorithm for the traffic grooming problem in ring networks.*

Proof. Recall that in the Minimum Weight Set Cover problem, each subset S_i has an associated weight, $weight[S_i]$. The weight of a cover is the sum of the individual weights of its sets.

Let w be the coloring returned by $GROOMBYSC(k)$ and w^* an optimal coloring. We will use the shortcut $\#ADM^*$ for $\#ADM^{w^*}$.

On one hand

$$\begin{aligned}\#ADM^w &= \sum_\lambda \#ADM^w_\lambda = \sum_\lambda endpoints(PART_\lambda) \\ &\leq \sum_\lambda endpoints(S_\lambda) = \sum_\lambda weight[S_\lambda] = weight(SC). \end{aligned} \quad (1)$$

On the other hand GREEDYSC is an H_f-approximation algorithm, where f is the maximum cardinality of the sets in the input. In our case $f = g \cdot k$. In other words if SC^* is a minimum weight set cover on the set S, we have

$$weight(SC) \leq H_{g \cdot k}\ weight(SC^*). \quad (2)$$

Clearly if $\overline{SC}$ is an arbitrary set cover of S, by definition

$$weight(SC^*) \leq weight(\overline{SC}). \quad (3)$$

Combining the inequalities (1), (2) and (3) we get

$$\#ADM^w \leq H_{g \cdot k}\ weight(\overline{SC})$$

for any set cover $\overline{SC}$ of S.

In the following claim we will show the existence of a set cover $\overline{SC}$ satisfying $weight(\overline{SC}) \leq\ \#ADM^* \left(1 + \frac{2g}{k}\right)$, which implies

$$\#ADM^w \leq\ \#ADM^*\ H_{g \cdot k} \left(1 + \frac{2g}{k}\right). \qquad \square$$

Claim. There exists a set cover $\overline{SC}$ of S, such that $weight(\overline{SC}) \leq\ \#ADM^* \left(1 + \frac{2g}{k}\right)$.

Proof. Let $w^*(P) = \{1, 2, ..., W^*\}$ and $1 \leq \lambda \leq W^*$. Consider the set V^*_λ of nodes v such that $ADM^*_\lambda(v) = 1$, namely having an ADM operating at wavelength λ at node v. We divide V^*_λ into sets of k nodes starting from an arbitrary node and going clockwise along the cycle (see Figure 1). Let $V_{\lambda,j}$ be the subsets of nodes obtained in this way. Let

$$\#ADM^*_\lambda = |V^*_\lambda| = kq_\lambda + r_\lambda \quad (4)$$

where $r_\lambda = |V^*_\lambda| \mod k$ and $0 \leq r_\lambda < k$.

Clearly $\forall 1 \leq j \leq q_\lambda, |V_{\lambda,j}| = k$, and in case $r_\lambda > 0$ we have $|V_{\lambda,q_\lambda+1}| < k$. In both cases $|V_{\lambda,j}| \leq k$. Therefore, each $V_{\lambda,j}$ is considered in the outer loop of phase 1 of the algorithm, and hence, is added to S.

For $V_{\lambda,j}$ we define $\overline{S}_{\lambda,j}$ to be the set of paths in $P^{w^*}_\lambda$ having their counter-clockwise endpoint in $V_{\lambda,j}$. As $V_{\lambda,j}$ has at most k nodes, and every node may be the clockwise endpoint of at most g paths from a $1-$colorable set, we have

$\left|\overline{S}_{\lambda,j}\right| \leq g \cdot k$. Therefore, $\overline{S}_{\lambda,j}$ is considered by the algorithm in the inner loop of phase 1. Being 1-colorable it should be added to S, thus $\overline{S}_{\lambda,j} \in S$.

Every $p \in P_{\lambda}^{w^*}$ has its both endpoints in the sets $V_{\lambda,j}$. In particular, it has its clockwise endpoint in $V_{\lambda,j}$ for a certain j, thus it is an element of some $\overline{S}_{\lambda,j}$. Therefore $\overline{SC}_{\lambda} \stackrel{def}{=} \cup_j \left\{\overline{S}_{\lambda,j}\right\}$ is a cover of $P_{\lambda}^{w^*}$. Considering all colors $1 \leq \lambda \leq W^*$ we conclude that $\overline{SC} \stackrel{def}{=} \cup_{\lambda=1}^{W*} \overline{SC}_{\lambda}$ is a cover of P.

Therefore $\overline{SC}$ is a cover of P with sets from S. It remains to show that its weight has the claimed property.

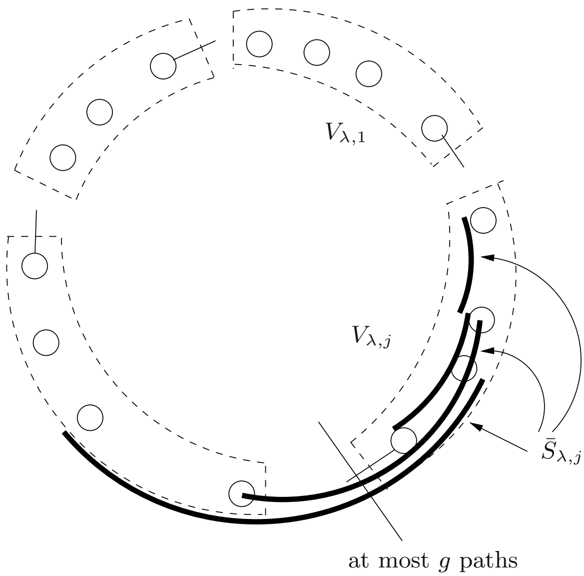

Fig. 1. The sets $V_{\lambda,j}$ and $\overline{S}_{\lambda,j}$ ($k = 4$)

Summing up equation (4) over all possible values of λ we obtain $\#ADM^* = k\sum_{\lambda} q_{\lambda} + \sum_{\lambda} r_{\lambda}$, which implies:

$$\sum_{\lambda} q_{\lambda} \leq \frac{\#ADM^*}{k} \tag{5}$$

We claim that $\forall j \leq q_{\lambda}, weight[\overline{S}_{\lambda,j}] = endpoints(\overline{S}_{\lambda,j}) \leq k+g$. This is because:

– The endpoints of the paths with both endpoints in $\overline{S}_{\lambda,j}$ are in $V_{\lambda,j}$ and $|V_{\lambda,j}| = k$.
– The number of paths having only the clockwise endpoint in set $V_{\lambda,j}$ is at most g. This follows from the observation that these paths should use the unique edge in the clockwise cut of $V_{\lambda,j}$. As the set $\overline{S}_{\lambda,j}$ is 1-colorable, the number of these paths is at most g.

For the set $j = q_\lambda + 1$ (which exists only if $r_\lambda > 0$) the above bound becomes $weight(\overline{S}_{\lambda,q_\lambda+1}) \leq r_\lambda + g \cdot q_\lambda$. This is because:

- The endpoints of the paths with both endpoints in $\overline{S}_{\lambda,q_\lambda+1}$ are in $V_{\lambda,q_\lambda+1}$ and $|V_{\lambda,q_\lambda+1}| = r_\lambda$.
- By the same argument as before, the paths having only the clockwise endpoint in $V_{\lambda,q_\lambda+1}$ are at most g in number. When $q_\lambda \geq 1$, $g \leq g \cdot q_\lambda$ and we are done. Otherwise $q_\lambda = 0$ meaning that $V_{\lambda,1}$ is the unique set. Then the number of paths having exactly one endpoint in the set is zero.

Summing up for all $1 \leq j \leq q_\lambda + 1$ we get:

$$weight(\overline{SC}_\lambda) \leq \sum_{j=1}^{q_\lambda}(k+g)+r_\lambda+g\cdot q_\lambda = (k+g)q_\lambda+r_\lambda+g\cdot q_\lambda = kq_\lambda+r_\lambda+2g\cdot q_\lambda$$

Summing up for all λ and recalling (4) and (5) we get:

$$\begin{aligned} weight(\overline{SC}) &= \sum_\lambda weight(\overline{SC}_\lambda) \leq \sum_\lambda (kq_\lambda + r_\lambda + 2g\cdot q_\lambda) = \#ADM^* + 2g\sum_\lambda q_\lambda \\ &\leq \#ADM^* + 2g\frac{\#ADM^*}{k} = \left(1+\frac{2g}{k}\right)\#ADM^*. \end{aligned}$$

□

Theorem 1. *There is a $2\ln g + o(\ln g)$-approximation algorithm for the traffic grooming problem in ring networks.*

Proof. The approximation ratio ρ of GROOMBYSC(k) is at most $H_{g\cdot k}\left(1+\frac{2g}{k}\right)$.

We substitute $k = g \ln g$ and get:

$$\begin{aligned} \rho &\leq H_{g^2 \ln g}\left(1+\frac{2}{\ln g}\right) \leq (1+\ln(g^2\ln g))\left(1+\frac{2}{\ln g}\right) \\ &= (1+2\ln g+\ln\ln g)\left(1+\frac{2}{\ln g}\right) = 2\ln g + o(\ln g) \end{aligned}$$

□

5 Discussion and Open Problems

We presented an approximation algorithm for ring networks, whose approximation ratio is $2\ln g + o(\ln g)$. Note that the approximation ratio of any algorithm for this problem is between 1 and $2g$.

Our algorithm can be used in arbitrary networks. In some topologies the analysis will yield a similar result. For this, note that the only point in the analysis that used the fact that the topology is a ring is where we considered the unique edge between the blocks of an optimal solution. Therefore a similar analysis follows for any topology and set of demands in which any solution can be partitioned in a similar way. This clearly includes all graphs which consists of blocks $B_0, B_1, ..., B_b$ whose sizes are bounded by $\alpha \leq k$ (k is the parameter used in our analysis) and at most $\beta = O(1)$ edges connecting consecutive blocks B_i and $B_{i+1 \mod b}$.

We mention few open problems which arise from this study.

– Improve the analysis of algorithm $GROOMBYSC(k)$.
– Find an algorithm with a better performance guarantee.
– Analyze algorithm $GROOMBYSC(k)$ for general topology and set of requests.

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