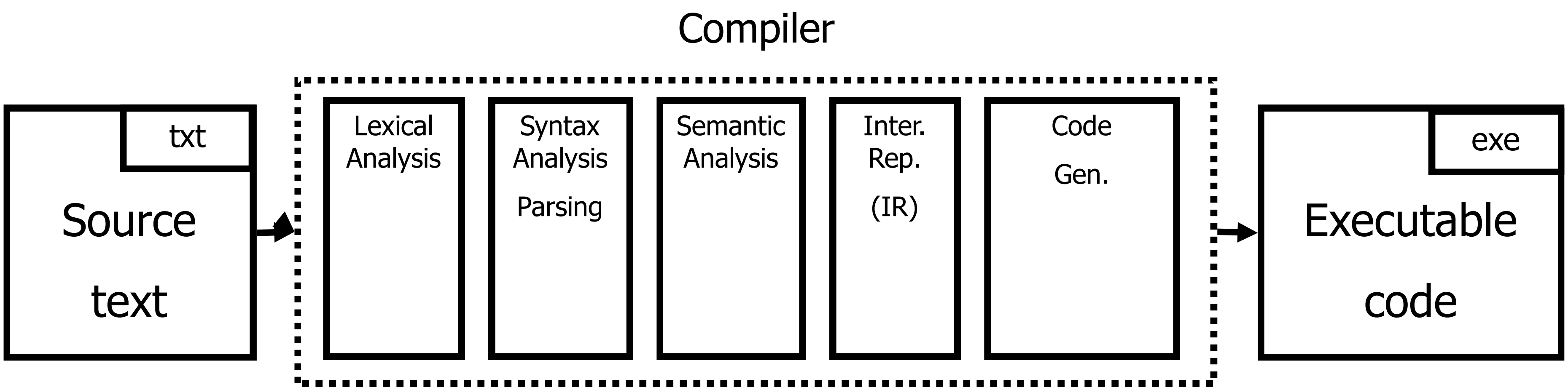
Lecture 03 – Syntax analysis: top-down parsing

THEORY OF COMPILATION

Eran Yahav

You are here


Compiler
txt
Source
text
Lexical
Analysis
Syntax
Analysis
Parsing
Semantic
Analysis
Inter.
Rep.
(IR)
Code
Gen.
exe
Executable
code

Last Week: from characters to tokens

txt

x = b*b – 4*a*c

Token Stream

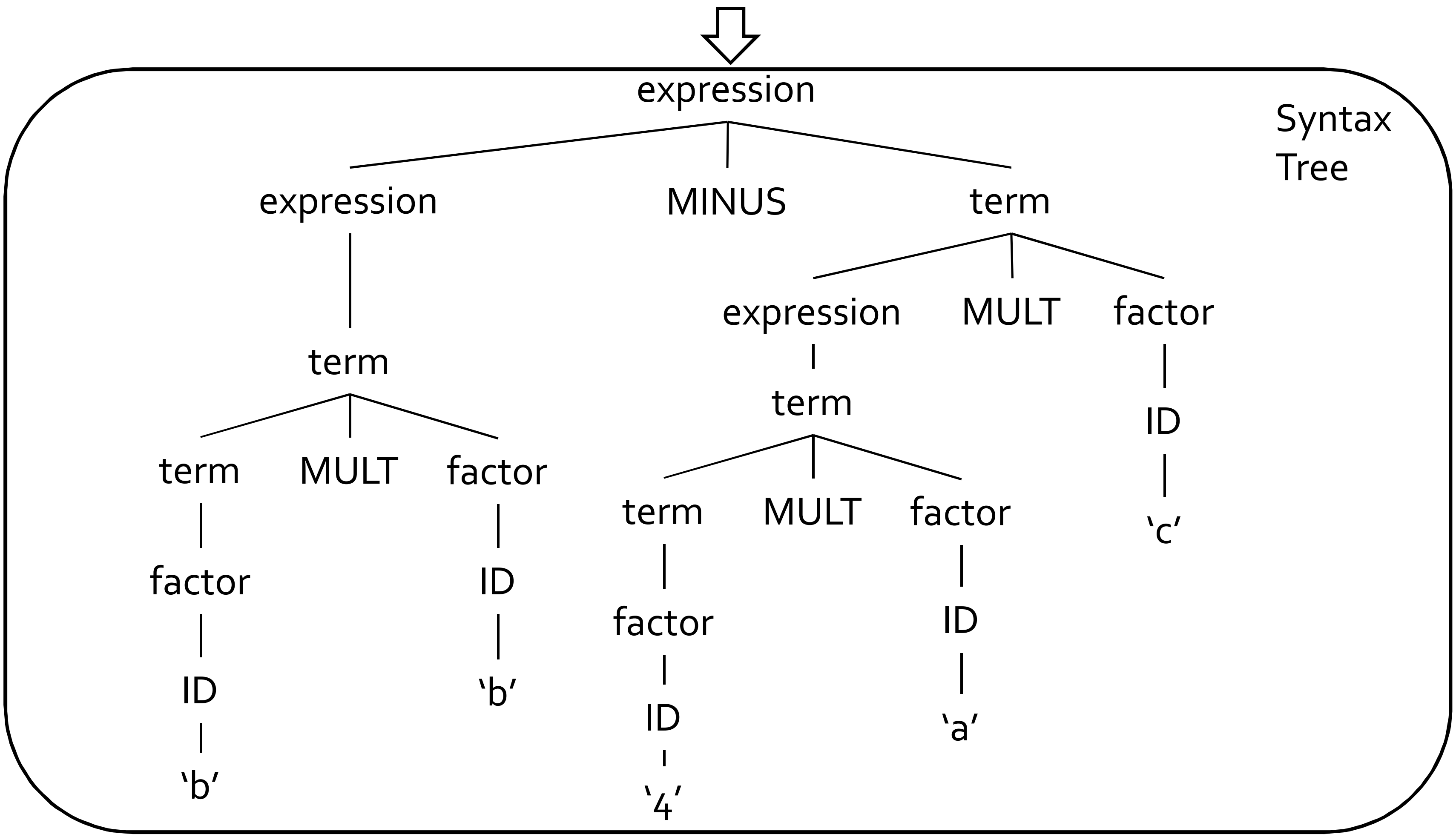
<ID,”x”> <EQ> <ID,”b”> <MULT> <ID,”b”> <MINUS> <INT,4> <MULT> <ID,”a”> <MULT> <ID,”c”>

Last Week: Regular Expressions

Basic Patterns	**Matching**
x	The character x
.	Any character, usually except a new line
[xyz]	Any of the characters x,y,z
Repetition Operators	
R?	An R or nothing (=optionally an R)
R*	Zero or more occurrences of R
R+	One or more occurrences of R
Composition Operators	
R1R2	An R1 followed by R2
R1|R2	Either an R1 or R2
Grouping	
(R)	R itself

Today: from tokens to AST

<ID,”x”> <EQ> <ID,”b”> <MULT> <ID,”b”> <MINUS> <INT,4> <MULT> <ID,”a”> <MULT> <ID,”c”>



Lexical Analysis | Syntax Analysis | Sem. Analysis | Inter. Rep. | Code Gen.

Parsing

- Goals
  - Is a sequence of tokens a valid program in the language?
  - Construct a structured representation of the input text
  - Error detection and reporting

- Challenges
  - How do you describe the programming language?
  - How do you check validity of an input?
  - Where do you report an error?

Context free grammars

$$G = (V,T,P,S)$$

- V – non terminals
- T – terminals (tokens)
- P – derivation rules
  - Each rule of the form $V \longrightarrow (T \cup V)^*$
- S – initial symbol

Why do we need context free grammars?

$$S \longrightarrow SS$$

$$S \longrightarrow \mathbf{(}S\mathbf{)}$$

$$S \longrightarrow \mathbf{()}$$

Example

$$S \longrightarrow S;S$$

$$S \longrightarrow id := E$$

$$E \longrightarrow id \mid E + E \mid E * E \mid (E)$$

V = { S, E }
T = { id, '+', '*', '(', ')'}

Derivation

input

x := z;
y := x + z

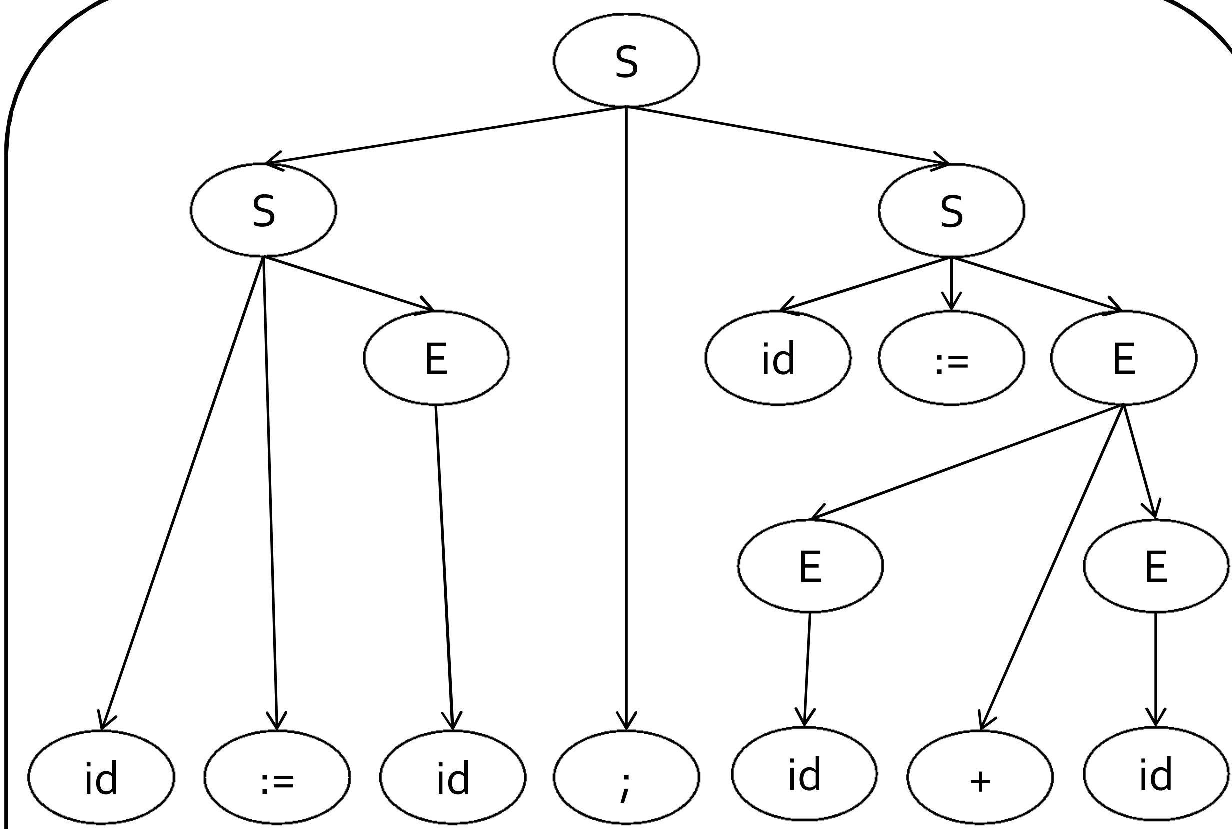
grammar

S → S;S
S → id := E
E → id | E **+** E | E ***** E | **(** E **)**

```
              S
               -------------------------------- S → S;S
          S   ;  S
           ------------------------------------ S → id := E
id := E   ;  S
           ------------------------------------ E → id
id := id ;  S
                 ------------------------------ S → id := E
id := id ;  id := E
                       ------------------------ E → E + E
id := id ;  id := E + E
                           -------------------- E → id
id := id ;  id := E + id
                       ------------------------ E → id
id := id ;  id := id + id
   x:= z  ;   y := x  + z
```

Parse Tree

```
        S

    S      ;   S

id := E  ;   S

id := id ;   S

id := id ; id := E

id := id ; id := E + E

id := id ; id := E + id

id := id ; id := id + id

x:= z  ; y := x  + z
```



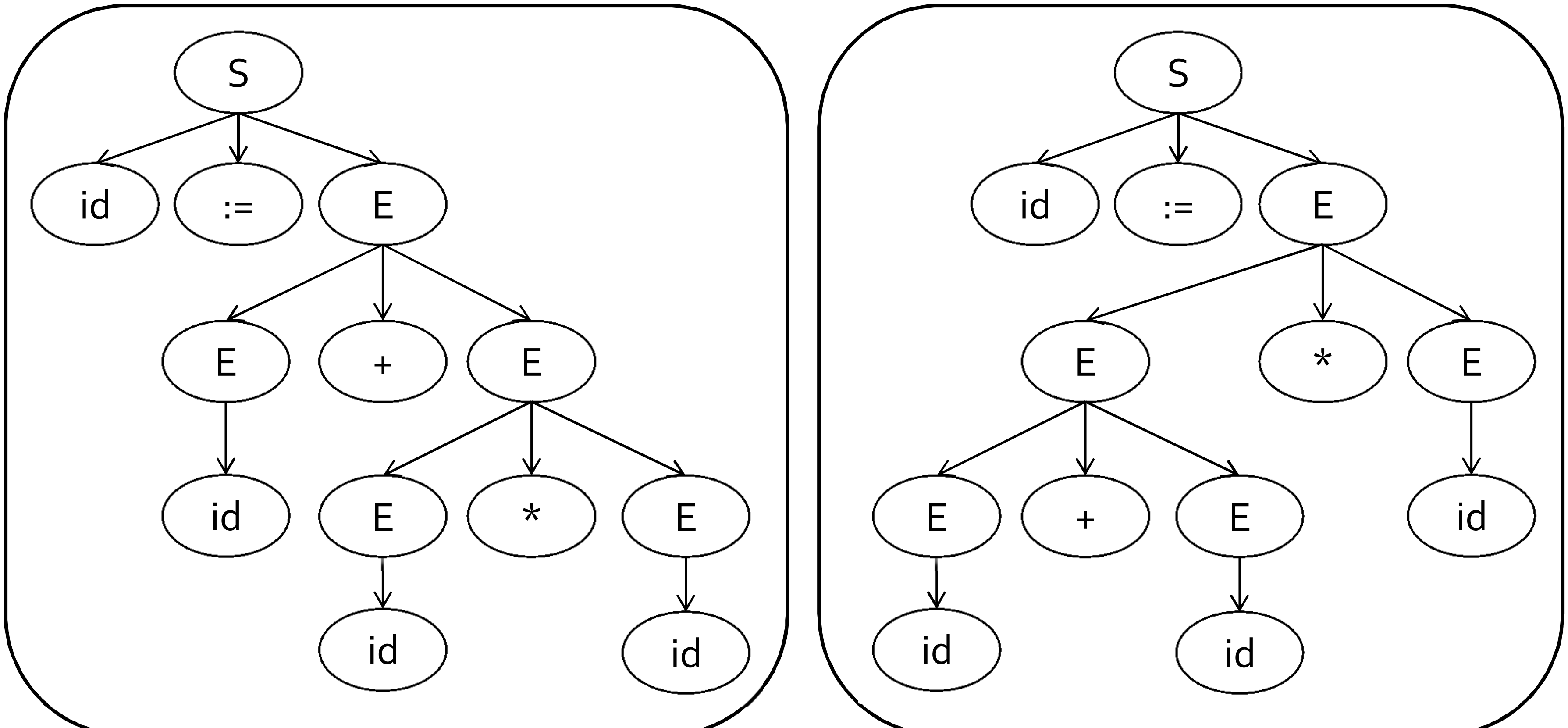
Questions

- How did we know which rule to apply on every step?
- Does it matter?
- Would we always get the same result?

Ambiguity

x := y+z*w

S → S;S
S → id := E
E → id | E **+** E | E ***** E | **(** E **)**



Leftmost/rightmost Derivation

- Leftmost derivation
  - always expand leftmost non-terminal
- Rightmost derivation
  - Always expand rightmost non-terminal

- Allows us to describe derivation by listing the sequence of rules
  - always know what a rule is applied to

- Orders of derivation applied in our parsers (coming soon)

Leftmost Derivation

x := z;
y := x + z

```
S → S;S
S → id := E
E → id | E + E | E * E | ( E )
```

```
             S
              ------------------------------  S → S;S
         S  ;  S
         ---------------------------------  S → id := E
id := E  ;  S
         ---------------------------------  E → id
id := id ;  S
              ----------------------------  S → id := E
id := id ;  id := E
                   -------------------  E → E + E
id := id ;  id := E + E
                   -------------------  E → id
id := id ;  id := id + E
                        --------------  E → id
id := id ;  id := id + id
   x:= z  ;   y := x  + z
```

Rightmost Derivation

x := z;
y := x + z

```
S → S;S
S → id := E
E → id | E + E | E * E | ( E )
```

```
              S
                ------------------------------  S → S;S
          S   ;  S
                   --------------------------   S → id := E
          S   ;  id := E
                         --------------------   E → E + E
          S   ;  id := E + E
                             ----------------   E → id
          S   ;  id := E + id
                         --------------------   E → id
          S   ;  id := id + id
          -----------------------------------   S → id := E
id := E   ;  id := id + id
          -----------------------------------   E → id
id := id  ;  id := id + id
   x:= z  ;   y := x  + z
```

Bottom-up Example

x := z;
y := x + z

```
S → S;S
S → id := E
E → id | E + E | E * E | ( E )
```

```
id := id ; id := id + id
        ---------------------------- E → id
 id := E;  id := id + id
   --------------------------------- S → id := E
        S ; id := id + id
                  ---------------- E → id
        S ; id := E + id
                        -------- E → id
        S ; id := E + E
                      ---------- E → E + E
        S ; id := E
                 --------------- S → id := E
        S ;  S
          -------------------------- S → S;S
          S
```

Bottom-up picking left alternative on every step → Rightmost derivation when going top-down

Parsing

- A context free language can be recognized by a non-deterministic pushdown automaton

- Parsing can be seen as a search problem
  - Can you find a derivation from the start symbol to the input word?
  - Easy (but very expensive) to solve with backtracking

- CYK parser can be used to parse any context-free language but has complexity $O(n^3)$

- We want efficient parsers
  - Linear in input size
  - Deterministic pushdown automata
  - We will sacrifice generality for efficiency

“Brute-force” Parsing

x := z;
y := x + z

```
S → S;S
S → id := E
E → id | E + E | E * E | ( E )
```

```
                id := id ; id := id + id

E → id                                   E → id

id := E;  id := id + id      id := id ;id := E+ id      ...

id := E;  id := id + id      id := E;  id := id + id
```

(not a parse tree… a search for the parse tree by exhaustively applying all rules)

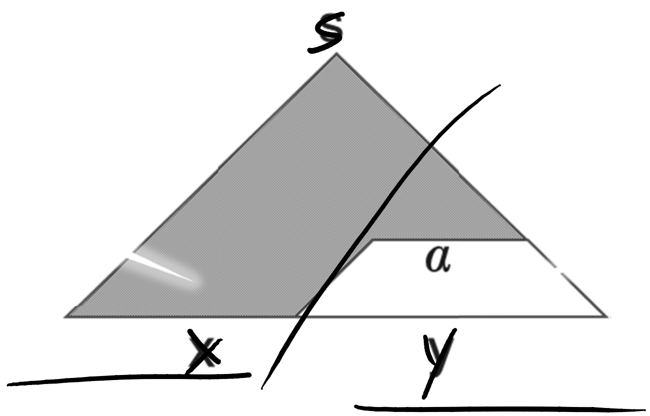
Efficient Parsers

- Top-down (predictive)
  - Construct the leftmost derivation
  - Apply rules "from left to right"
  - Predict what rule to apply based on nonterminal and token
- Bottom up (shift reduce)
  - Construct the rightmost derivation
  - Apply rules "from right to left"
  - Reduce a right-hand side of a production to its non-terminal

Efficient Parsers

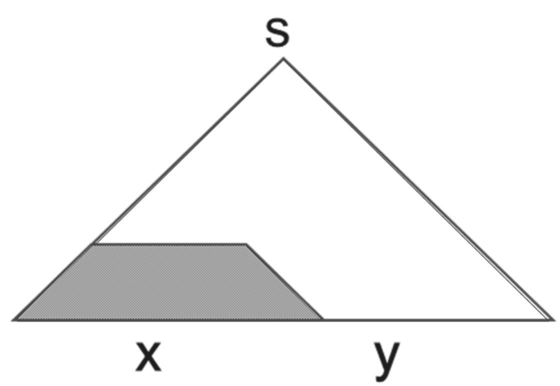
- Top-down (predictive parsing)

already read…

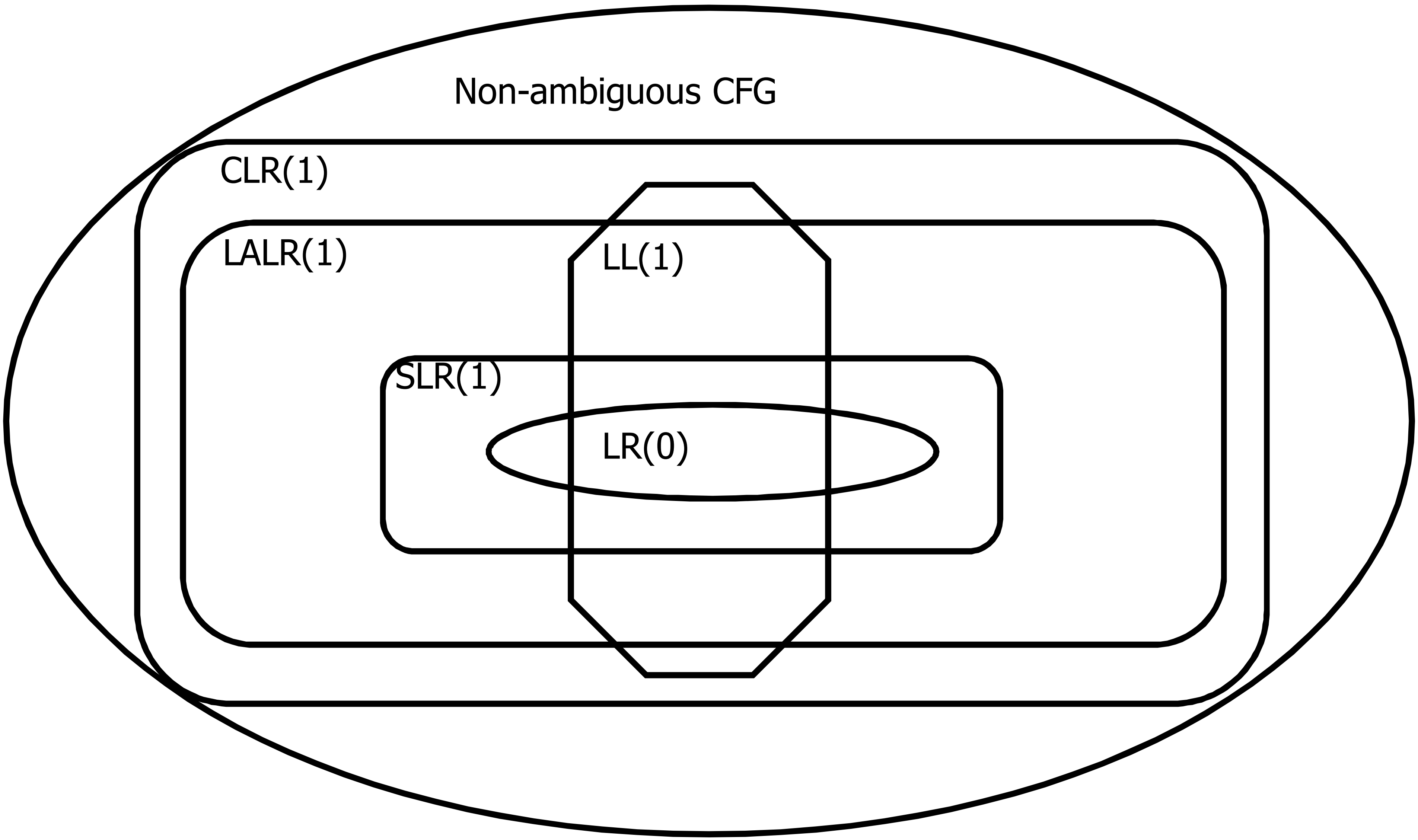


to be read…

- Bottom-up (shift reduce)



Grammar Hierarchy



Top-down Parsing

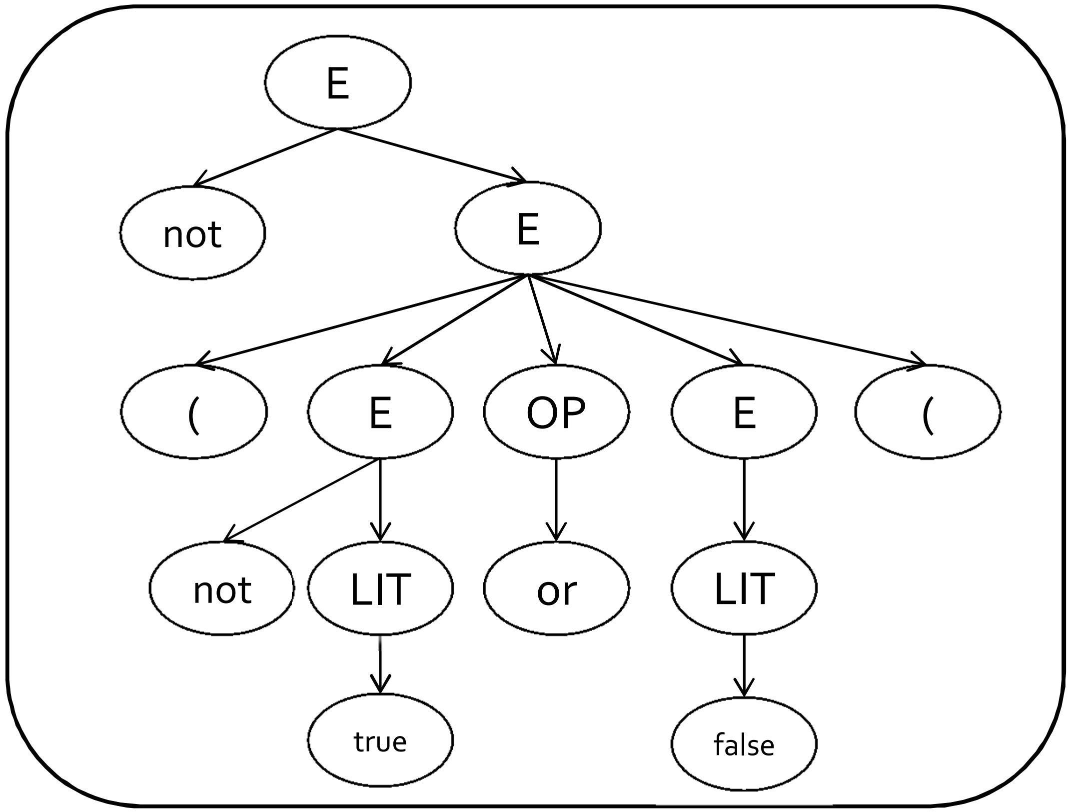
- Given a grammar G=(V,T,P,S) and a word w
- Goal: derive w using G
- Idea
  - Apply production to leftmost nonterminal
  - Pick production rule based on next input token
- General grammar
  - More than one option for choosing the next production based on a token
- Restricted grammars (LL)
  - Know exactly which single rule to apply
  - May require some lookahead to decide

Boolean Expressions Example

not (not true or false)

```
E → LIT | (E OP E) | not E
LIT → true | false
OP → and | or | xor
```

E =>
not E =>
not (E OP E) =>
not (not E OP E) =>
not (not LIT OP E) =>
not (not true OP E) =>
not (not true or E) =>
not (not true or LIT) =>
not (not true or false)



Production to apply is known from next input token

Recursive Descent Parsing

- Define a function for every nonterminal
- Every function work as follows
  - Find applicable production rule
  - Terminal function checks match with next input token
  - Nonterminal function calls (recursively) other functions
- If there are several applicable productions for a nonterminal, use lookahead

Matching tokens

```
void match(token t) {
  if (current == t)
    current = next_token();
  else
    error;
}
```

- Variable current holds the current input token

functions for nonterminals

```
E → LIT | (E OP E) | not E
LIT → true | false
OP → and | or | xor
```

```
void E() {
  if (current ∈ {TRUE, FALSE}) // E → LIT
    LIT();
  else if (current == LPAREN)  // E → ( E OP E )
   match(LPARENT); E(); OP(); E(); match(RPAREN);
  else if (current == NOT)    // E → not E
   match(NOT); E();
  else
   error;
 }

 void LIT() {
   if (current == TRUE) match(TRUE);
   else if (current == FALSE) match(FALSE);
   else error;
 }
```

functions for nonterminals

E → LIT
  | (E OP E)
  | not E
LIT → true
  | false
OP → and
  | or
  | xor

```
void E() {
        if (current ∈ {TRUE, FALSE})                    LIT();
        else if (current == LPAREN)                     match(LPARENT);
                                                        E(); OP(); E();
                                                        match(RPAREN);
        else if (current == NOT)                match(NOT); E();
        else error;
}
```

```
void LIT() {
        if (current == TRUE)                    match(TRUE);
        else if (current == FALSE)  match(FALSE);
        else                                    error;
}
```

```
void OP() {
        if (current == AND)                     match(AND);
        else if (current == OR)         match(OR);
        else if (current == XOR)        match(XOR);
        else                                    error;
}
```

Adding semantic actions

- Can add an action to perform on each production rule
- Can build the parse tree
  - Every function returns an object of type Node
  - Every Node maintains a list of children
  - Function calls can add new children

Building the parse tree

```
Node E() {
  result = new Node();
  result.name = “E”;
  if (current ∈ {TRUE, FALSE}) // E → LIT
    result.addChild(LIT());
  else if (current == LPAREN)  // E → ( E OP E )
    result.addChild(match(LPARENT));
    result.addChild(E());
    result.addChild(OP());
    result.addChild(E());
    result.addChild(match(RPAREN));
  else if (current == NOT)     // E → not E
    result.addChild(match(NOT));
    result.addChild(E());
  else error;
    return result;
}
```

Recursive Descent

```
void A() {
  choose an A-production, A -> X1X2…Xk;
  for (i=1; i≤ k; i++) {
    if (Xi is a nonterminal)
      call procedure Xi();
    elseif (Xi == current)
      advance input;
    else
       report error;
  }
}
```

- How do you pick the right A-production?
- Generally – try them all and use backtracking
- In our case – use lookahead

Recursive descent: are we done?

```
term → ID | indexed_elem
indexed_elem → ID [ expr ]
```

- The function for indexed_elem will never be tried…
  - What happens for input of the form
    - ID [expr]

Recursive descent: are we done?

```
S → A a b
A → a | ε
```

```
int S() {
  return A() && match(token('a')) && match(token('b'));
}
int A()  {
 return match(token('a')) || 1;
}
```

- What happens for input "ab" ?
- What happens if you flip order of alternatives and try "aab"?

Recursive descent: are we done?

$$E \rightarrow E - term$$

```
int E() {
  return E() && match(token('-')) && term();
}
```

- What happens with this procedure?
- Recursive descent parsers cannot handle left-recursive grammars

Figuring out when it works…

1

```
term → ID | indexed_elem
indexed_elem → ID [ expr ]
```

2

```
S → A a b
A → a | ε
```

3

```
E → E - term
```

3 examples where we got into trouble with our recursive descent approach

FIRST sets

- For every production rule $A \rightarrow \alpha$
  - FIRST(α) = all terminals that α can start with
  - i.e., every token that can appear as first in α under some derivation for α
- In our Boolean expressions example
  - FIRST(LIT) = { true, false }
  - FIRST((E OP E)) = { '(' }
  - FIRST (not E) = { not }

- No intersection between FIRST sets => can always pick a single rule

- If the FIRST sets intersect, may need longer lookahead
  - LL(k) = class of grammars in which production rule can be determined using a lookahead of k tokens
  - LL(1) is an important and useful class

FOLLOW Sets

- What do we do with nullable alternatives?
  - Use what comes afterwards to predict the right production
- For every production rule $A \longrightarrow \alpha$
  - FOLLOW(A) = set of tokens that can immediately follow A
- Can predict the alternative A_k for a non-terminal N when the lookahead token is in the set
  - FIRST(A_k) $\cup$ (if A_k is nullable then FOLLOW(N))

LL(k) Grammars

- A grammar is in the class LL(K) when it can be derived via:
    - Top down derivation
    - Scanning the input from left to right (L)
    - Producing the leftmost derivation (L)
    - With lookahead of k tokens (k)

- A language is said to be LL(k) when it has an LL(k) grammar

Back to our 1st example

```
term → ID | indexed_elem
indexed_elem→ ID [ expr ]
```

- FIRST(ID) = { ID }
- FIRST(indexed_elem) = { ID }

- FIRST/FIRST conflict

Left factoring

- Rewrite the grammar to be in LL(1)

```
term → ID | indexed_elem
indexed_elem→ ID [ expr ]
```

⇩

```
term → ID after_ID
after_ID → [ expr ] | ε
```

Intuition: just like factoring x*y + x*z into x*(y+z)

Left factoring – another example

```
S → if E then S else S
    | if E then S
    | T
```

⇩

```
S → if E then S S'
    | T
S' → else S | ε
```

Back to our 2nd example

```
S → A a b
A → a | ε
```

- FIRST(S) = { 'a' }, FOLLOW(S) = { }
- FIRST(A) = { 'a' ε }, FOLLOW(A) = { 'a' }

- FIRST/FOLLOW conflict

Substitution

```
S → A a b
A → a | ε
```

Substitute A in S

```
S → a a b | a b
```

Left factoring

```
S → a after_A
after_A → a b | b
```

Back to our 3rd example

```
E → E - term
```

- Left recursion cannot be handled with a bounded lookahead
- What can we do?

Left recursion removal

$N \rightarrow N\alpha \mid \beta$

$$\Rightarrow$$

$N \rightarrow \beta N'$
$N' \rightarrow \alpha N' \mid \varepsilon$

G1 G2

- L(G1) = β, βα, βαα, βααα, …
- L(G2) = same

- For our 3rd example:

```
E → E - term
```

$$\Rightarrow$$

```
E → term TE
TE → - term TE | ε
```

LL(k) Parsers

- Recursive Descent
  - Manual construction
  - Uses recursion

- Wanted
  - A parser that can be generated automatically
  - Does not use recursion

LL(k) parsing with pushdown automata

- Pushdown automaton uses
  - Prediction stack
  - Input stream
  - Transition table
    - nonterminals x tokens -> production alternative
    - Entry indexed by nonterminal N and token t contains the alternative of N that must be predicated when current input starts with t

LL(k) parsing with pushdown automata

- Two possible moves
  - Prediction
    - When top of stack is nonterminal N, pop N, lookup table[N,t]. If table[N,t] is not empty, push table[N,t] on prediction stack, otherwise – syntax error
  - Match
    - When top of prediction stack is a terminal T, must be equal to next input token t. If (t == T), pop T and consume t. If (t ≠ T) syntax error

- Parsing terminates when prediction stack is empty. If input is empty at that point, success. Otherwise, syntax error

Example transition table

(1) E → LIT
(2) E → (E OP E)
(3) E → not E
(4) LIT → true
(5) LIT → false
(6) OP → and
(7) OP → or
(8) OP → xor

Input tokens

Which rule should be used

Nonterminals

	(	**)**	**not**	**true**	**false**	**and**	**or**	**xor**	**$**
E	2		3	1	1				
LIT				4	5				
OP						6	7	8	

Simple Example

aacbb$

$A \rightarrow aAb \mid c$

Input suffix	Stack content	Move
aacbb$	A$	predict(A,a) = A ⟶ aAb
aacbb$	aAb$	match(a,a)
acbb$	Ab$	predict(A,a) = A ⟶ aAb
acbb$	aAbb$	match(a,a)
cbb$	Abb$	predict(A,c) = A ⟶ c
cbb$	cbb$	match(c,c)
bb$	bb$	match(b,b)
b$	b$	match(b,b)
$	$	match($,$) – success

	a	b	c
A	A ⟶ aAb		A ⟶ c

Simple Example

abcbb$

$A \rightarrow aAb \mid c$

Input suffix	Stack content	Move
abcbb$	A$	predict(A,a) = A ⟶ aAb
abcbb$	aAb$	match(a,a)
bcbb$	Ab$	predict(A,b) = ERROR

	a	b	c
A	A ⟶ aAb		A ⟶ c

Error Handling

- Mentioned last time
  - Lexical errors
  - Syntax errors
  - Semantic errors (e.g., type mismatch)

Error Handling and Recovery

x = a * (p+q * (-b * (r-s);

- Where should we report the error?
- The valid prefix property
- Recovery is tricky
  - Heuristics for dropping tokens, skipping to semicolon, etc.

Error Handling in LL Parsers

```
c$
```

$S \rightarrow a\ c \mid b\ S$

Input suffix	Stack content	Move
c$	S$	predict(S,c) = ERROR

- Now what?
  - Predict bS anyway "missing token b inserted in line XXX"

	a	**b**	**c**
S	$S \rightarrow a\ c$	$S \rightarrow bS$	

Error Handling in LL Parsers

c$

S → a c | b S

Input suffix	Stack content	Move
bc$	S$	predict(b,c) = S → bS
bc$	bS$	match(b,b)
c$	S$	Looks familiar?

- Result: infinite loop

	a	b	c
S	S → a c	S → bS	

Error Handling

- Requires more systematic treatment
- Enrichment
  - Acceptable-set method
  - Not part of course material

Summary

- Parsing
  - Top-down or bottom-up
- Top-down parsing
  - Recursive descent
  - LL(k) grammars
  - LL(k) parsing with pushdown automata
- LL(K) parsers
  - Cannot deal with left recursion
  - Left-recursion removal might result with complicated grammar

Coming up next time

- More syntax analysis