

# Quantum cryptography with coherent states

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The safety of a quantum key distribution system relies on the fact that any eavesdropping attempt on the quantum channel creates errors in the transmission. For a given error rate, the amount of information that may have leaked to the eavesdropper depends on both the particular system and the eavesdropping strategy. In this work, we discuss quantum cryptographic protocols based on the transmission of weak coherent states and present a system, based on a symbiosis of two existing systems, for which the information available to the eavesdropper is significantly reduced. This system is therefore safer than the two previous ones. We also suggest a possible experimental implementation.

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## I. INTRODUCTION

The only known method to exchange secret information through a communication channel in a proven secure way is to use the so-called one-time pad (for a good review of both classical and quantum cryptography, see [1]). In this technique, the data, which are represented by a string of bits, are combined with a random string of bits of equal length called the key, and are then sent through the communication channel. The randomness of the key ensures that the encoded message is also completely random and as such totally unintelligible to a potential eavesdropper. The safety of the transmission is thus entirely dependent on the safety of the key, which has to be secret and shared just by both legitimate users. Moreover, safety can be guaranteed only if the key is used once, and then discarded. The problem is therefore how to distribute the random key between users in a secure way. Classically, the only possibility is either through personal meetings, or through a trusted courier, which makes the technique rather expensive, and not practical for many applications. Therefore most practical cryptographic systems nowadays rely on different principles [1]. However, these cannot really *guarantee* the safety of the transmission, but rely on a weaker property of the system, namely, that it is *computationally* safe. This means that the system can be broken in principle, but that the computation time required to do so is too long to pose a real threat. The main problem with this approach is that its safety could be destroyed by technological progress (faster computers) or mathematical advances (faster algorithms or future theoretical progress in computation theory). Another technique, whose safety does not rely on computing abilities, and which was only recently developed, is quantum cryptography (for an introduction,

see [2]).

In quantum cryptography, the two users, generally referred to as Alice (the sender) and Bob (the receiver), have two kinds of communication channels at their disposal. One is a classical public channel, which can be overheard by anybody, but cannot be modified; and the second is a quantum channel, whose main characteristic is that any attempt at eavesdropping will create errors in the transmission. The quantum channel will be used to transmit the secret key, and the classical public channel will be used to exchange information and to send the encoded message. In principle, this is sufficient to ensure the safety of a transmission: Alice and Bob exchange a series of bits over the quantum channel, and then use part of the transmission to test for eavesdropping. If they find any discrepancy between their strings, they can infer that an eavesdropper, usually referred to as Eve, was listening and that their transmission is not secret. If they detect no errors, they can assume that the key is safe. By testing a large proportion of their initial string, they can attain any safety level they wish. Unfortunately, quantum channels are very sensitive devices, and due to the imperfections of the channels and of the detectors, some errors will always be unavoidable. The problem facing Alice and Bob is therefore, for a given error rate, to estimate the amount of information that may have leaked to Eve, and decide on the safety of the transmission. This of course depends both on the particular system used by Alice and Bob, and on the eavesdropping strategy adopted by Eve. A safer system is a system for which the amount of information that may have leaked to Eve is lower. If the information leaked to Eve is not too high, Alice and Bob can use classical information processing techniques to reduce it to approximately zero, at the expense of shortening their strings [3–5].

At present, there exist three different quantum cryptographic systems. The first one relies on the transmission of single photons randomly polarized along four directions [6]. As single photons are difficult to produce experimentally, a slight modification of this system, using weak pulses instead of single photons, was the first one to be implemented in practice [3,7,8]. The second system, which is conceptually the simplest, uses only two nonorthogonal quantum states [9]. Its implementation relies on weak coherent pulses, with a phase difference between them [9,10]. The third system is based on the creation of pairs of Einstein-Podolsky-Rosen (EPR) correlated photons [11]. One of its potential advantages is that the correlations are between single photons and not weak pulses, which can be a great advantage, as we shall emphasize later. However, creation and transmission over long distances of EPR correlated pairs is technologically more difficult, and it is not clear yet whether this will prove practical [12]. In this work, we focus on quantum cryptographic schemes implemented with weak pulses of coherent light. We compare the safety of the first and the second of these quantum cryptographic systems, and present an alternative system, which is a symbiosis of both, and for which the safety can be significantly increased.

In Sec. II, we analyze the first system, referred to as the four-states system. In Sec. III, we turn to the second one, named the two-states system, and present an alternative implementation. We introduce our 4+2 system in Sec. IV, and show that it is more sensitive to eavesdropping than the two previous ones. In Sec. V, we show the dangers associated with a lossy transmission line, and conclude in Sec. VI.

## II. FOUR-STATES PROTOCOL

### A. Principle of four-states systems

This protocol was developed by Bennett and Brassard [6]. The sender, Alice, chooses at random one out of four states, e.g., for polarized photons  $\uparrow$ ,  $\leftrightarrow$ ,  $\nearrow$ , or  $\nwarrow$ , and sends it to the receiver, Bob. The two states  $\uparrow$  and  $\nearrow$  stand for bit value 0, while the other two,  $\leftrightarrow$  and  $\nwarrow$ , stand for 1. Bob chooses, also at random, a basis,  $\oplus$  or  $\otimes$ , in which he measures the polarization. When his basis corresponds to Alice's, his bit should be perfectly correlated with hers, whereas when his basis is the conjugate, there is no correlation between his result and Alice's original choice. By discussing over the public channel, Alice and Bob agree to discard all the instances where they did not use the same basis (half of the total on average). The result is what we call the sifted key, which should be two perfectly correlated strings, but which may contain errors. The two fundamental properties of this protocol are

(i) the choice of basis is completely hidden from the other protagonist (the two bases correspond to the same density matrix), as well as from any mischievous eavesdropper, Eve;

(ii) when Alice and Bob use different bases, there is no correlation between their bits.

The first one, (i), ensures that, as the eavesdropper Eve cannot know which basis was used, she will unavoidably introduce errors. The second one, (ii), is not really necessary, but is preferable, as it reduces the information available to Eve to a minimum [13] (25% for each photon on which she eavesdropped). There have been various works analyzing eavesdropping strategies, calculating the information available to Eve as a function of the error rate, and developing information processing techniques to reduce it to any required level [3–5,14].

### B. Implementation with weak pulses

One technical difficulty with this scheme is that in principle it should be implemented by means of single photons [6]. As these are difficult to generate experimentally, existing schemes rely on weak pulses of coherent or thermal light, with much less than one photon per pulse on average [3,7,8]. This ensures that the probability of having two or more photons in a pulse remains very small. This strategy reduces the transmission rate (recent experiments use about one-tenth of a photon per pulse), while providing no advantage to an honest participant. More precisely, if Alice and Bob use coherent pulses  $|\alpha\rangle$ , the transmission rate  $t^{(4)}$  is given by

$$t^{(4)} \equiv \frac{1}{2} \left( 1 - |\langle \alpha|0\rangle|^2 \right) = \frac{1}{2} \left( 1 - e^{-|\alpha|^2} \right), \quad (1)$$

where the factor 1/2 comes from the fact that half of the transmissions had to be discarded (when Alice and Bob used different bases). In fact, even for these weak pulses, the probability of having two or more photons per pulse may not always be neglected (for the above pulses, one out of 20 nonzero pulses will have two photons). We shall show in Sec. V how to take this into account.

### C. Safety of four-states systems

In order to get quantitative results, we shall assume that Eve uses the intercept-resend strategy: she intercepts the pulses, attempts to gain as much information as possible, and sends to Bob a new pulse, prepared according to the information she obtained. Moreover, we shall assume that she eavesdrops in the bases used by Alice and Bob,  $\oplus$  or  $\otimes$ . This is the intercept-resend strategy which provides her with the most information on the sifted key [5]. However, it is not yet known whether this is the optimal strategy. It is easy to see that, when Eve eavesdrops on a fraction  $\eta$  of the transmissions, the error rate created is  $\eta/4$  (when Eve uses the correct basis, she does not introduce any error, while she creates a 50% error rate when she uses the wrong basis), and that the information she obtained is  $\eta/2$  (she has total information when she used the correct basis, and none when she used the wrong one). Moreover, the scheme is completely symmetric, so that Eve shares the same information with Alice and with Bob. Therefore we can write the mutual

information shared by Alice and Eve and shared by Eve and Bob as a function of the error rate  $Q$ :

$$I_{AE}^{(4)}(Q) = I_{EB}^{(4)}(Q) = 2Q. \quad (2)$$

In this system, the intensity of the weak pulses, or equivalently the transmission rate  $t^{(4)}$  defined in (1), has no influence. In the following, we will compare (2) to the information obtained by Eve for the other two systems.

### III. TWO-STATES PROTOCOL

#### A. Principle of two-states systems

In this protocol [9], Alice chooses between only two nonorthogonal states, and sends one to Bob. As these are not orthogonal, there is no way for Bob to decode them deterministically. However, by means of a generalized measurement, also known as *positive operator valued measure* or POVM [15,16], he can perform a test which will sometimes fail to give an answer, and at all other times give the correct one. In essence, instead of having a binary test (with results 0 or 1), which will unavoidably create errors when the two states are not orthogonal, Bob has a ternary system, with possible results 0, 1, or ? (with ? corresponding to inconclusive results). For example, if Alice sends a 0, Bob may get either a 0 or an inconclusive result, but he will never get a 1. We present a practical implementation of such a POVM in the next section (for a theoretical description, see [15,16]). As with the previous system, Alice and Bob use their public channel to discard all the inconclusive results. They should now have two perfectly correlated strings, except for possible errors. The safety of the protocol against eavesdropping is ensured by the fact that Eve cannot get deterministic results either. She may attempt to get as much information as possible by projecting the states onto an orthogonal basis. This is discussed in Sec. III C, where we show that this will unavoidably create errors. Another possibility for Eve is to mimic Bob's measurement, and obtain deterministic results on a fraction of the bits. However, in this case, she will have to guess Alice's choice on the remaining bits, and this will provide her with less information than the previous strategy. Let us emphasize that, as we want Eve's guesses to result in errors on Bob's side, this scheme should not be implemented with weak pulses only. In this case, Eve could simply intercept the transmission, resend a pulse to Bob only when she managed to obtain the bit sent by Alice (i.e., 0 or 1), and send nothing when she obtained an inconclusive result. The signature of eavesdropping would then just be a reduction in the transmission rate. In the original proposal, this is overcome by using phase encoding of a weak pulse (the two states are  $|\pm\alpha\rangle$ , with overlap  $e^{-2|\alpha|^2}$ ), which is sent together with a strong pulse, used also as a phase reference [9,10]. When Eve fails to obtain information, she still has to send the strong pulse, which will create errors in the reception. Ekert *et al.* [16] recently analyzed various eavesdropping strategies related to this system. Here, we shall restrict ourselves to the

intercept-resend strategy and compare the safety of various schemes.

#### B. Alternative implementation

Let us now suggest an alternative implementation of the two-states system, which will be later extended to our  $4 + 2$  protocol. We shall call it *parallel reference* implementation, to distinguish it from the *sequential reference* implementation of [9]. Alice uses weak coherent states, with phase encoding 0 or  $\pi$  with respect to a strong coherent state. We denote the weak states by  $|\pm\alpha\rangle$  and the strong state by  $|\beta\rangle$ . The overlap between the two states is given by:

$$|\langle\alpha| - \alpha\rangle| \equiv \cos\delta = e^{-2|\alpha|^2}. \quad (3)$$

Instead of sending the two states one after the other, Alice uses two orthogonal polarizations:  $|\pm\alpha\rangle$ , say, will have vertical polarization, and  $|\beta\rangle$  will have horizontal polarization. The decoding on Bob's side is schematized in Fig. 1. The two states are separated by a polarization beam splitter (PBS).  $|\beta\rangle$  is rotated to vertical polarization and sent through a mainly transmitting beam splitter (BS1) to detector D1. A small fraction of  $|\beta\rangle$ , equal to  $|\alpha\rangle$ , is sent to interfere with  $|\pm\alpha\rangle$  at BS2, and towards two detectors D2 and D3. A count in D2, say, corresponds to phase zero, while a count in D3 corresponds to  $\pi$ . No count in both D2 and D3 is of course an inconclusive result. It is easy to calculate the probability of such a result:

$$\text{Prob}(?) = e^{-2|\alpha|^2}, \quad (4)$$

which is equal to the overlap between the two states. It was shown that (4) represents the optimum for separating deterministically two nonorthogonal states [17]. Since all the cases corresponding to inconclusive results have to be discarded, the transmission rate of the channel is given by

$$t^{(2)} \equiv 1 - \text{Prob}(?) = 1 - e^{-2|\alpha|^2}. \quad (5)$$

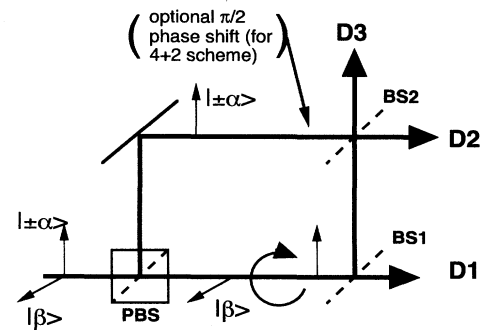


FIG. 1. Schematic of the detection system for two-states system and for  $4+2$  system. The description is given in the text. The optional  $\pi/2$  phase shift is used for the  $4+2$  system, and corresponds to the choice of decoding basis.

Detector D1 should always fire, and can therefore be used as a trigger for the other two, enabling them only for a short time corresponding to the length of the pulse (this will reduce the dark counts). Moreover, an eavesdropper with an inconclusive result would still have to send  $|\beta\rangle$ , which will result in random counts in D2 and D3. A small modification of the above will be used for our 4 + 2 protocol.

### C. Safety of the two-states systems

As in Sec. II, we assume that Eve performs an intercept-resend strategy, and attempts to get as much information as possible on the state sent by Alice. This is obtained by projecting it onto the orthogonal basis  $B_{\text{sym}} = (i, j)$  as shown in Fig. 2 [15], and provides Eve with probabilistic information only. Another method for Eve would be to use the same POVM as Bob. This would provide her with a smaller amount of deterministic information. The error rate introduced by Eve is equal to the probability of making the wrong reading, e.g., the probability of obtaining  $j$  when the input was in fact  $|\alpha\rangle$ :

$$q = \text{Prob}(j/\alpha) = \frac{1 - \sin \delta}{2}, \quad (6)$$

where  $\delta$  is defined in (3) and shown in Fig. 2. With this particular eavesdropping strategy, the transmission channel Alice-Eve is known as a binary symmetric channel, and is fully characterized by the error rate  $q$  [18]. The mutual information shared by Alice and Eve is equal to the channel capacity and is given by [16]

$$i_{AE}(\delta) = 1 + \frac{1 - \sin \delta}{2} \log_2 \left( \frac{1 - \sin \delta}{2} \right) + \frac{1 + \sin \delta}{2} \log_2 \left( \frac{1 + \sin \delta}{2} \right) \quad (7)$$

$i_{AE}(\delta)$  is the maximum information that can be extracted from two states with overlap  $\cos \delta$ . On the other hand, after Bob discards all his inconclusive results, the transmission channel Eve-Bob is perfect. Therefore the mutual information Eve-Bob is

$$i_{EB} = 1. \quad (8)$$

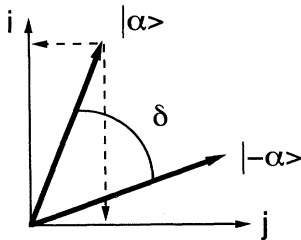


FIG. 2. Representation of two nonorthogonal states in the Hilbert space. The maximum information that can be extracted from them is obtained by projecting them onto the basis  $B_{\text{sym}} = (i, j)$ . A positive projection on  $i$  is read as input state  $|\alpha\rangle$ , and reciprocally for  $j$ . However, as the projection of  $|\alpha\rangle$  on  $j$  is nonzero, this creates errors in the detection.

This strategy, where Eve eavesdrops on all the transmitted photons, will create a high error rate. In order to reduce it, Eve shall only eavesdrop on a fraction  $\eta$  of the transmissions, thus creating an error rate

$$Q \equiv \eta q = \eta \frac{1 - \sin \delta}{2}. \quad (9)$$

It is now easy to obtain the mutual information between Alice and Eve,  $I_{AE}^{(2)}$ , and between Eve and Bob,  $I_{EB}^{(2)}$ , as a function of the error rate  $Q$  and the angle  $\delta$  between the states:

$$I_{AE}^{(2)}(Q, \delta) = \frac{2Q}{1 - \sin \delta} i_{AE}(\delta), \quad (10)$$

$$I_{EB}^{(2)}(Q, \delta) = \frac{2Q}{1 - \sin \delta}. \quad (11)$$

Using (3) and (5), we can also write the mutual information as a function of the transmission rate for each value of the error rate. These functions are plotted in Fig. 3, where the information gained for the four-states system is used as a reference.

## IV. 4 + 2 PROTOCOL

### A. Principle and motivation

The basic idea behind this protocol is that the four-states scheme does not require the two states in each basis to be orthogonal. The safety of this scheme relies entirely on the two points (i) and (ii) mentioned in Sec. II. Any scheme using two pairs of states (each pair corresponding to one nonorthonormal basis) satisfying both (i) and (ii) is an equally good candidate for a four-states protocol. Moreover, by choosing nonorthogonal states in each pair, we will get the additional advantage of the two-states protocol, namely, that Eve cannot differentiate deterministically between the two states in each basis. As we show

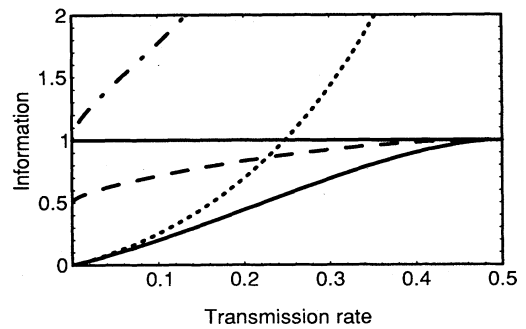


FIG. 3. Mutual information shared by Alice and Eve (full and dotted curves) and by Eve and Bob (dashed and dash-dotted curves) as a function of the transmission rate (in bit/pulse). The dotted and dash-dotted curves refer to the two-states system (Sec. III), and the full and dashed ones to the 4+2 system (Sec. IV). They are normalized with respect to the mutual information for the four-states system (Sec. II). This figure illustrates the advantage of the 4+2 protocol, since the information Eve may get is smaller, for any given transmission rate.

in the following, this will make the scheme more sensitive to eavesdropping. A second advantage, shared with the two-states system, is that this scheme will be more resilient in the case of a lossy transmission line. This aspect will be discussed in Sec. V. A graphic comparison between the different systems is given in Fig. 4, where we plot the various states on the Poincaré sphere.

### B. Implementation

Let us apply this to our previous parallel reference phase encoding scheme. The first pair will correspond to 0 and  $\pi$  phase shifts, while the second pair will correspond to  $\pi/2$  and  $3\pi/2$ . Explicitly, the four states are  $|\alpha\rangle$ ,  $|\alpha\rangle$ ,  $|\alpha\rangle$ , and  $|\alpha\rangle$ . Let us emphasize that these states satisfy condition (i) only when the intensity is low enough, since pulses containing two or more photons do not satisfy it. Indeed, if we take into account the two-photon-number state, it is easy to see that the two pairs  $|\pm\alpha\rangle$  and  $|\pm\alpha\rangle$  are not even in the same two-dimensional (2D) Hilbert space, and therefore cannot correspond to the same density matrix. In the following, we shall assume that this condition is verified. We shall, however, lift this restriction in Sec. V, where we present an analysis for a lossy transmission line.

The detection system is similar to the one explained in Sec. III, with the addition of an optional  $\pi/2$  phase shifter in one arm of the interferometer (see Fig. 1). When Bob wants to measure in the first basis, he does not put the phase shift, and his detection scheme will therefore differentiate between  $|\alpha\rangle$  and  $|\alpha\rangle$ . When he wants to measure in the second basis, he puts the  $\pi/2$  phase shift, so that his detection scheme will now differentiate between  $|\alpha\rangle$  and  $|\alpha\rangle$ . Of course, when Bob uses the wrong basis (say Alice sent state  $|\alpha\rangle$  and Bob puts the  $\pi/2$  phase shift), Bob's result is totally uncorrelated with Alice's choice. In many instances Bob will not get any count in his detectors D2 and D3, which correspond to inconclusive results, and will be discarded.

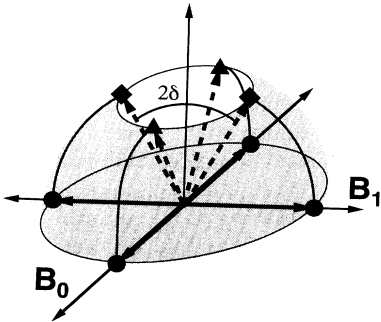


FIG. 4. Difference between the different systems shown by plotting the states on the Poincaré sphere. The circles represent the four-states system, where the two states in basis  $B_0$  (or  $B_1$ ) are orthogonal, which corresponds to opposite directions on the Poincaré sphere. The squares (or equivalently the triangles) represent the two-states system, where the states are not orthogonal, and therefore cannot be distinguished deterministically. The 4+2 system makes use of both the triangles (basis  $B_0$ ) and the squares (basis  $B_1$ ).

### C. Safety

The problems facing Eve are now twofold. She does not know the basis used by Alice (as in the four-states method), which means that even when she has a conclusive result, she cannot be sure that it is relevant. Moreover, as the states are not orthogonal, she is faced with two possibilities: either to try to get deterministic results, which means that in many instances she will get no information at all, but will still have to make a decision on which state to send to Bob; or to try to get probabilistic information, in which case she will know very little on each bit. In both cases, she will introduce errors, before she even starts to deal with the basis. This is the main difference with the usual four-states scheme: Alice and Bob make use of the fact that they send weak pulses and not single photons to enhance the safety.

In order to compare the various schemes, we first need to calculate the transmission rate. Since in half of the cases, Alice and Bob will use different bases, and therefore will have to discard the transmission, we easily get

$$t^{(4+2)} = \frac{1 - \text{Prob}(?) }{2} = \frac{1 - e^{-2|\alpha|^2}}{2} = \frac{1 - \cos \delta}{2}. \quad (12)$$

As in Sec. II, we now assume that Eve eavesdrops on a fraction  $\eta$  of the transmissions, and uses the intercept-resend strategy in the bases used by Alice and Bob. When she uses the wrong basis, which happens on a fraction  $\eta/2$ , she does not get any information, but still creates an error rate  $1/2$  as in the four-states system. When she uses the correct bases, again on a fraction  $\eta/2$ , the system reduces to the two-states system. Following Sec. III, we assume that Eve attempts to get as much probabilistic information as possible. The error rate is thus given by (6) and the information gain by (7). The overall error rate is therefore

$$Q = \frac{\eta}{2} \left( 1 - \frac{\sin \delta}{2} \right), \quad (13)$$

and the information gained by Eve

$$I_{AE}^{(4+2)}(Q, \delta) = \frac{Q}{1 - \frac{\sin \delta}{2}} i_{AE}(\delta), \quad (14)$$

$$I_{EB}^{(4+2)}(Q, \delta) = \frac{Q}{1 - \frac{\sin \delta}{2}}. \quad (15)$$

The comparison between the three systems is given in Fig. 3, where we plot the various information as functions of the transmission rate, and use the four-states system as a reference. Note that, as the transmission rate is different for the two-states system and the 4+2 system, we compare systems with different values of  $\alpha$  (or, equivalently, with different values of the overlap  $\cos \delta$ ). With the type of eavesdropping strategy considered here, the information available to Eve grows linearly with the error rate for all the systems. Therefore, as the plot is normalized with respect to the four-states system, there is no dependence on  $Q$ . Figure 3 shows a clear advantage for the two-states system with respect to the four-

states system at very low transmission rates. However, the information leaked to Eve increases rapidly with the transmission rate. In contrast, the 4+2 system is always preferable to both other systems. Let us emphasize that our analysis is in principle correct for any value of the overlap  $\cos \delta$  between the states, if the various states remain in a 2D Hilbert space. However, this is not the case for the suggested implementation with weak coherent states. In this case, the analysis is only appropriate for very weak states, where the probability of having more than one photon is sufficiently small, and therefore where the overlap between the states is always close to 1. This is likely to be correct in practical applications, but it is still important to present the corrections to the analysis when two-photon states become relevant. This is the task of Sec.V.

## V. LOSSY TRANSMISSION LINE

In the above work, we have analyzed the relative sensitivity of various schemes to eavesdropping, under the assumption that there was at most one photon per pulse. However, when we use pulses of coherent or thermal light, there is a nonzero probability to have more than one photon in a pulse, even for very weak pulses. When the transmission line between Alice and Bob is approximately lossless, the fact that a few pulses may have more than one photon is not too damaging, but only gives some free information to an eavesdropper. However, when the transmission line is lossy, using weak pulses can prove disastrous for Alice and Bob. Let us assume that the transmission line is a silica optical fiber, which is always slightly lossy (about 0.2 dB/km). If we want to use the cryptographic system over reasonable distances, say up to 50 km, transmission losses will be as high as 10 dB, or about 90%. An eavesdropper with superior technology could replace the fiber by a perfectly transparent one, and use the excess power for her mischievous purposes.

For the four-states system based on polarization (Sec. II), this can be very damaging. Indeed, as polarization and photon number are independent variables, there is no problem in principle in selecting the few pulses with two or more photons and separating them into two one-photon pulses, without changing the polarization. Eve could then send one of the two pulses to Bob, while keeping the other one. She would measure her signal only after Alice and Bob have disclosed their bases, which means that she will get complete information on these pulses. If the pulses sent by Alice are coherent and contain on average 1/10 photon, Eve would obtain about one such pulse out of 200, and would send all of these to Bob without attenuation. Remembering that Bob only expects to receive one photon out of 100 pulses sent by Alice, we see that Eve can know the polarization of half of the pulses received by Bob, without creating any error. For thermal light, where the photon distribution is wider (one pulse out of 100 would contain more than one photon), Eve would actually know the polarization of all the pulses. This can of course be reduced by using even weaker pulses, but this in turn would lower the transmis-

sion rate and the signal to noise ratio (mainly because of the dark counts of the detectors). This shows that a combination of weak pulses with high channel loss is a deadly combination for this cryptographic system. If the losses are high enough, Eve can get full information on the transmission, and this without causing a single error. Let us emphasize, however, that any system using only single photons would be insensitive to the above attack.

In contrast to the four-states system, both the two-states systems based on phase encoding (Sec. III), and our 4+2 protocol (Sec. IV) will be more resilient against this attack. The main reason is that, as phase and number are conjugate variables, any attempt by Eve to select the two-photon pulses will unavoidably randomize the phase of the pulse. Therefore the best Eve can do is to split the incoming pulses, and send to Bob the weaker pulses through a lossless fiber. For the 90% losses mentioned above, Eve can therefore split 90% of the pulses. Bob, of course, has no way of discovering that the losses are now due to Eve. The malicious Eve can now keep her pulses, wait for Alice and Bob to disclose the basis they used, and then try to extract the maximum information from them. In contrast to the four-states system, her information is limited by the fact that the states are not orthogonal. In fact, the maximum information that she can extract is given by  $i_{AE}(\delta)$  defined in (7). For the above example, with 1/10 photon per pulse on average and 90% losses, Eve has pulses with 0.09 photons at her disposal. The maximum information that she can extract is then about 0.23 bits. In a practical scheme, this would have to be added to the information available to Eve as a function of the error rate. Let us emphasize that Eve can adopt this beam splitting strategy even in the case of four-states systems with weak pulses. However, as was shown in the previous paragraph, in this case she can also use the independence of polarization and photon number to get more information.

## VI. CONCLUSION

The rule of the game in quantum cryptography is that the honest participants are limited by the available technology, while the mischievous eavesdropper is only confined by the laws of quantum mechanics. This is in order to ensure that the safety of the schemes does not depend on future technological advances, no matter how unlikely. At present, one of the main technological limitations is the difficulty of generating single photons. For this reason, most existing schemes rely on weak pulses, which are easy to generate. However, in practical implementations [3,7,8], this fact was considered to be only a limitation. The fact that the pulses had a strong vacuum component, or equivalently the fact that the two pulses in the same basis were not orthogonal, was not utilized by the users. In this work, we presented a system which made use of this vacuum component to enhance the safety of the transmission.

The reason why the task of the eavesdropper is made more difficult with this system, is that it combines the strengths of the two systems it is built from: as in the

four-states system, Eve does not know the basis used by Alice and Bob; as in the two-states system, she cannot distinguish with certainty between the two nonorthogonal states. It is worth noting that the usual implementation of the four-states system with weak pulses actually satisfies these two criteria. The weak pulses with orthogonal polarizations are *not* orthogonal in the Hilbert space, due to their common vacuum component. The ingredient which is missing from these systems is the use of a strong reference pulse in the detection, which ensures that Eve has to send a signal to Bob, even when she could not get any information on the signal sent by Alice. When this is added to the system, the use of weak pulses becomes an advantage for the legitimate users. Indeed, one experimental realization of quantum cryptography based on an interferometric scheme [7] is somewhat similar to our suggested implementation (see Fig. 1). The main difference between the two schemes is that the authors of [7] do not use a strong reference pulse (in their setup,  $|\beta|^2 = |\alpha|^2$ , and the beam splitter BS1 is replaced by a mirror). Therefore, as emphasized above, their scheme does not take advantage of the nonorthogonality of the weak pulses. Whenever she obtains an unsatisfactory re-

sult in her attempt to eavesdrop, Eve may simply block the transmission and send nothing to Bob. A slight modification of the existing experiment could therefore provide a much safer scheme.

Unfortunately, even very weak pulses have a nonzero probability of having more than one photon. This creates the most acute problem when the two legitimate users have a lossy transmission line. In this case, we showed that the usual polarization-based scheme appears to be impractical, as too much information may leak to an eavesdropper, without causing any error. The two other schemes, which combine nonorthogonal states, phase encoding, and a reference pulse, appear to be less sensitive to this problem, as Eve cannot get total information, no matter what the losses are. However, following the results of Sec. V, it is clear that, when the transmission line between Alice and Bob is very lossy, single-photon transmission would be preferable.

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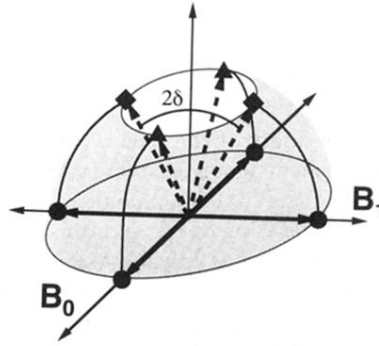


FIG. 4. Difference between the different systems shown by plotting the states on the Poincaré sphere. The circles represent the four-states system, where the two states in basis  $B_0$  (or  $B_1$ ) are orthogonal, which corresponds to opposite directions on the Poincaré sphere. The squares (or equivalently the triangles) represent the two-states system, where the states are not orthogonal, and therefore cannot be distinguished deterministically. The 4+2 system makes use of both the triangles (basis  $B_0$ ) and the squares (basis  $B_1$ ).