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Analysis of Zeno Behaviors in Hybrid Systems

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Abstract

This is a substantially abbreviated version of Technion CIS Report CIS-2002-03, available at the first author's web page http://www.cs.technion.ac.il/~ heymann/. We investigate conditions for existence of Zeno behaviors in hybrid systems.

Keywords: Hybrid systems, Zenoness, control

1 Introduction

In recent years, various algorithms have been proposed for the synthesis of controllers for hybrid systems [1], [2], [3], [4], [5], [6], [9], [10], [11]. However, sometimes the synthesized controllers may force the system to undergo an unbounded (infinite) number of discrete configuration changes (switches) in a finite length of time and then violate the constraints. This phenomenon is called Zenoness[2] (or a Zeno behavior), and can be thought of as a type of instability of hybrid systems that constitutes a major impediment to "proper" system behavior, and is an obstacle to successful controller synthesis, even in cases when controllers actually exist.

With the aim of bypassing the difficulties created by the Zenoness phenomenon, several researchers proposed controller synthesis approaches, that limit the maximal switching rate of the synthesized controller, thereby yielding controlled systems that switch configurations at or below a specified upper rate. Such switching rate limitation is accomplished by imposing various structural constraints on either the system or on the controller [2], [3], [5], [11]. Yet, while such approaches guarantee that a synthesized controller will never yield a Zeno system, they do not answer the basic questions associated with the Zenoness phenomenon. Specifically, when controllers with the imposed switching rate constraint exist, are they necessarily minimally interventive for the system when no switching rate constraints are imposed? When controllers with the imposed switching rate constraint do not exist, what conclusions can be drawn regarding the existence and nature of controllers for the unconstrained system? Are Zeno behaviors inherently possible in the unconstrained system? When a safety controller for the constrained system exists, does there also exist a minimally interventive controller for the unconstrained one? If the answer to this latter question is affirmative, how are the two controllers related?

To answer these questions, we must investigate conditions for existence of Zeno behaviors in hybrid systems. We begin our investigation by examining *constant rate* systems in which each of the dynamic (state) variables has a constant rate in every discrete configuration. In the full version of the paper (http://www.cs.technion.ac.il/~ heymann/), we extend our investigation to *bounded rate* systems where the rate of each state variable is specified to lie within constant upper and lower bounds and to more general hybrid systems with nonlinear dynamics. There, we also use these results to investigate the existence and synthesis of controllers for hybrid systems.

Our approach is based on a simple but crucial observation that a state of the hybrid system is reachable at a given time if and only if it is reachable at the same time in an "equivalent" continuous system that is obtained as a suitable weighted combination of the dynamic equations of the hybrid system in the different discrete configurations. Thus, instead of a difficult investigation of the rather complicated class of behaviors of the hybrid system, we examine the very simple class of behaviors of the "equivalent" continuous system.

2 The Hybrid Machine Model

We use the hybrid-machine formalism as described e.g. in [7] to model a hybrid system. A *hybrid machine* is denoted by

$$HM = (Q, \Sigma, D, E, I, (q_0, x_0)).$$

The elements of HM are as follows. Q is a finite set of (discrete) configurations. We shall assume that the system has n configurations; that is, $dim(Q) = n$. Σ is a finite set of event labels. An event is an *input* event, denoted by $\underline{\sigma}$ (underlined), if it is received by the HM from its environment; and an *output* event, denoted by $\overline{\sigma}$ (overlined), if it is generated by the HM and transmitted to the environment. $D = \{d_q : q \in Q\}$ is the dynamics of the HM. For configuration i, the dynamics given by $\dot{x} = f_i(x, u)$, where x and u are, respectively, the state and input variables of appropriate dimensions. $E = \{(q, G \wedge \underline{\sigma} \rightarrow \overline{\sigma'}, q') : q, q' \in Q\}$ is a set of edges, where q is the configuration exited, q' is the configura-

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[2]After the Greek philosopher Zeno whose famous paradox about the race between Achillis and the turtle resembles the said behavior.

tion entered, $\underline{\sigma}$ is the input event, and $\overline{\sigma'}$ the output event. G is the guard. An edge $(q, G \wedge \underline{\sigma} \rightarrow \overline{\sigma'}, q', x^0_{q'})$ is interpreted as follows: If the guard G is true and the event $\underline{\sigma}$ is received as an input, then the transition to q' takes place at the instant $\underline{\sigma}$ is received. The output event $\overline{\sigma'}$ is transmitted as a side-effect at the same time. $I = \{I_q : q \in Q\}$ is a set of invariants. (q_0, x_0) denotes the initialization condition: q_0 is the initial configuration, and x_0 is the initial value of x. A more detailed description of the hybrid machine model can be found in the full version of the paper.

A *run* of the HM is a sequence

$q_0 \xrightarrow{e_1,t_1} q_1 \xrightarrow{e_2,t_2} q_2 \xrightarrow{e_3,t_3} \ldots$

where e_i is the ith transition and $t_i (\geq t_{i-1})$ is the time when the ith transition takes place. For each run, we define its trajectory, time stamp and path as follows.

The trajectory of the run is the sequence of the vector time functions of the (state) variables:

$x_{q_0}, x_{q_1}, x_{q_2}, \ldots$

where $x_{q_i} = \{x_{q_i}(t) : t \in [t_i, t_{i+1})\}$. The time stamp of the run is a (column) vector function $In(t), t \geq 0$, where $dim(In(t)) = dim(Q)$. If at time $t \geq 0$ HM is in the ith configuration, then $In(t)$ has value 1 in its ith entry and zeros in all others.

The path of the run is the sequence of the configurations. We say that a path is irreducible if for any two consecutive configurations q, q' in the sequence, q and q' have different dynamics ($d_q \neq d_{q'}$). A run is irreducible if its associated path is irreducible.

We shall call a run of a HM *dynamic* if all its transitions are dynamic transitions (triggered by guards becoming true). Every dynamic run can be reduced to an irreducible one. An unbounded irreducible dynamic run

$q_0 \xrightarrow{e_1,t_1} q_1 \xrightarrow{e_2,t_2} q_2 \xrightarrow{e_3,t_3} \ldots$

is called a *Zeno* run if

$lim_{i \rightarrow \infty} t_i = T < \infty$

A HM is called *Zeno* if it possesses Zeno runs. Otherwise it is called *non-Zeno* or *viable*. A hybrid machine all of whose runs are Zeno is called *strongly Zeno*.

Clearly Zeno HMs are ill defined, in that they may uncontrollably execute an unbounded number of transitions in a finite (and bounded) time interval and thus describe systems whose lifetime is limited, contrary to our intention of modeling ongoing behaviors (that never terminate). In the next sections we shall explore conditions under which hybrid machines possess Zeno behaviors.

3 Zenoness

In the simplified hybrid machine model described above, we assume that state variables are the same for all configurations. Such hybrid systems are called *homogeneous*.

For a run that starts at the initial state $x(0) = x_0$, the dynamics of $x(t)$ for $t \geq 0$ can then be expressed as

$$\dot{x} = F(x, u, t) = [f_1(x, u) \ \ldots \ f_n(x, u)] In(t). \quad (1)$$

This description, which resembles the dynamic representation of a continuous system, will be used below to derive various results on Zenoness.

To illustrate some aspects of the Zeno phenomenon, let us examine the following example.

Consider the hybrid system shown in Figure 1(a).

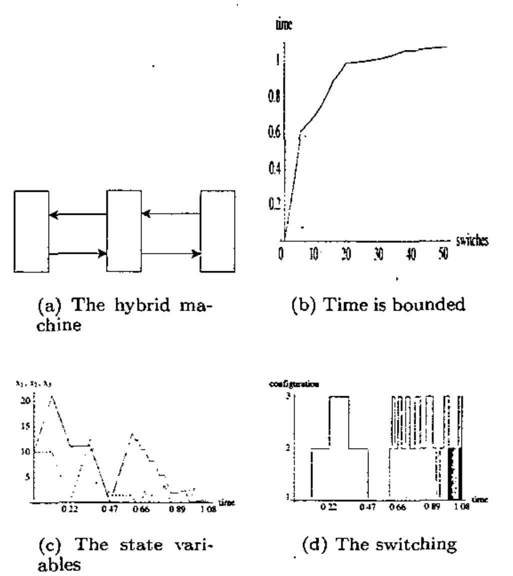


Figure 1: Example of a Zeno system

It consists of three configurations labeled by 1, 2, and 3. There are three continuous variables x_1, x_2, and x_3. The rates of changes of these variables are displayed in each configuration (thus, in configuration 1, $\dot{x_1} = 100$, $\dot{x_2} = -90$, $\dot{x_3} = 1$, etc.). When a variable reaches some lower bound[3] and the corresponding guard becomes true , a dynamic transition is triggered that takes the system to a different configuration (e.g., when x_2 becomes zero in configuration 1, a transition is triggered to configuration 2) as shown in Figure 1(a).

Note that in each configuration of the system, at least one variable is decreasing and will eventually cause the system to change configuration. We call such a variable an *active variable.*

This example is an extension of the two water-tank example that we first proposed in [7] and was later used by others [8]. However, the behavior of this system is much more complex than the two water-tank example,

[3]Without loss of generality, we assume that the lower bounds are 0 in this paper.

as can be seen in Figure 1. It is not very straightforward to deduce intuitively from the dynamics whether the system is Zeno. Indeed, the switching among the three configurations is highly irregular as shown by the simulation results in Figure 1(d) and the "water level" in each tank (the value of the variables) does not show an obvious pattern as can be seen in Figure 1(c). However, as can be seen in Figure 1(b), "time converges", that is, an unbounded number of transitions takes place in bounded time and hence the system is Zeno.

We are motivated, by this simple example and many others, to investigate the complex phenomenon of Zenoness.

To examine the Zenoness phenomenon, we review the concept of *instantaneous configuration cluster* (ICC) [7]. Let v be a valuation of the state vector and let q be a configuration. Suppose that q is entered by a dynamic transition guarded by G, whose value is true at v. Assume further that q has an outgoing dynamic transition guarded by G', which is also true at v. Since G' follows G instantaneously, we say that the transition associated with G' is triggered by that associated with G. A sequence of transitions $G_1, G_2, \dots$ is triggered by v if G_1 is true at v and G_{i+1} is triggered by G_i for all $i \geq 1$. For a given v, consider all transition sequences in the HM triggered by v. Let HM(v) denote the HM obtained by deleting all transitions that are not elements of transition sequences triggered by v. A strongly connected component (SCC)[4] of HM(v) that consists of two or more configurations is called an ICC. The triggering value v of the state vector will be called a *Zeno point* of the HM. Note that there may exist more than one ICC for a given Zeno point and there may be more than one Zeno point for an ICC. In the above example, $v = x = [0,0,0]$ is a Zeno point associated with an ICC which includes configurations 1, 2 and 3.

In [7] it is shown that existence of a Zeno point and its associated ICC is a necessary condition for Zenoness, although it is not sufficient. Clearly, once at a Zeno point, the behavior of the HM is necessarily Zeno. Thus, the question that must be examined is whether if initialized outside (or away from) a Zeno point, a possible run will enter the Zeno point after a bounded length of time. We shall say that a Zeno point is a *Zeno attractor* whenever there exist initializations of the HM outside the Zeno point such that for some run, the Zeno point will be reached in bounded time. Clearly, a HM is non-Zeno if and only if it has no Zeno attractor. Thus, the problem of checking Zenoness of a HM consists of identifying its ICCs, if any, and checking whether they include Zeno attractors. Since identifying ICCs is an easy job, in this paper, we address the latter issue.

We consider homogeneous hybrid systems with n configurations and m continuous variables. We confine our attention first to *constant rate* hybrid systems, for which the continuous dynamics in configuration j, $j = 1, 2, \dots, n$, is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} k_{1j} \\ k_{2j} \\ \dots \\ k_{mj} \end{bmatrix},$$

where the k_{ij}s are constant, and we shall consider systems that satisfy the following assumption:

Assumption 1

1. The legal region of the system is the nonnegative orthant $\mathbb{R}_+^m = \{x \in \mathbb{R}^m : x_i \geq 0, i = 1, 2, ..., m\}$.

2. All the system's configurations are in an ICC with respect to the Zeno point $x = 0$.

3. Every variable is active in some configurations.

4. In every configuration, there is at least one active variable.

5. In a given configuration, a unique transition is associated with each active variable x_i. This transition is triggered either by an event (generated by a controller) or by the associated guard $[x_i \leq 0]$ becoming true. Each transition leads the system to a configuration where the triggering variable x_i is not active.

In the above Assumption, (1) implies that a variable is active if and only if its derivative is negative, (2) states that every configuration is relevant to the Zeno behavior, (3) states that every variable is relevant to the Zeno behavior of the system, (4) ensures that the hybrid system cannot stay in any configuration indefinitely and hence the system is forced to perform an unbounded number of transitions over an unbounded interval of time, and (5) states that the hybrid system can be forced to exit a configuration at any time before $[x_i \leq 0]$ becomes true.

Let us consider a run of a hybrid system HM initialized at state $x(0) = x_0$. We assume that x_0 is in $int(\mathbb{R}_+^m)$, the interior of $\mathbb{R}_+^m$. Using equation (1), we obtain the state $x(t)$ at $t \geq 0$ as

$$x(t) = \int_0^t KIn(\tau)d\tau + x_0, \tag{2}$$

where K is the rate matrix

$$K = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & & & \\ k_{m1} & k_{m2} & \dots & k_{mn} \end{bmatrix}.$$

Equation 2 can be rewritten as

$$x(t) = \int_0^t KIn(\tau)d\tau + x_0 = Kt\alpha(t) + x_0, \tag{3}$$

[4] An SCC is a set of configurations for which there is a directed path from any configuration to any other.

where $\alpha(t) = \frac{1}{t}\int_0^t In(\tau)d\tau =: [\alpha_1(t)\ \alpha_2(t)\ ...\ \alpha_n(t)]'$. Note that $\alpha_i(t) \geq 0, i = 1, 2, ..., n$, and $\alpha_1(t) + \alpha_2(t) + ... + \alpha_n(t) = 1$. Thus, $\alpha_i(t)$ represents the fraction of time (up to time t), that the HM resides in configuration $i; i = 1, 2, ..., n$. In other words,

$$\alpha(t) \in \mathcal{A} := \{\alpha \in \mathbb{R}_+^n | \sum_{i=1}^{n} \alpha_i = 1\}.$$

It is readily noted that $x(t) = \int_0^t KIn(\tau)d\tau + x_0$ is also the solution of the following constant rate dynamical system

$$\begin{cases} \dot{x} = K\alpha \\ x(0) = x_0 \end{cases} \quad (4)$$

for $\alpha = \alpha(t)$. This much simpler "equivalent" system will serve us below to investigate the Zenoness properties of the hybrid system HM. In particular, we will show that the existence of Zenoness is closely related to the existence of solutions to the inequality $K\alpha \geq 0, \alpha \in \mathcal{A}$.

We shall make use of the following simple observation.

Lemma 1 Let HM be a homogeneous constant rate hybrid system satisfying Assumption 1 with initial state $x(0) = x_0 \in int(\mathbb{R}_+^m)$. Let $x \in int(\mathbb{R}_+^m)$ be any point. Then there exists a run of HM reaching x with a trajectory wholly contained in $\mathbb{R}_+^m$ if and only if for some $\alpha \in \mathcal{A}$ there exists a solution to system (4) starting at x_0 and reaching x. Moreover, in that case, the time T at which HM reaches x (i.e., $x(T) = x$) is the same as the time at which the equivalent system (4) reaches x.

The proofs of all the lemmas and theorems can be found in the full version of the paper.

By investigating the equivalent system (4) instead of the original hybrid system HM, we can simplify the problem of determining Zenoness significantly. In particular, we have the following necessary and sufficient condition for strong Zenoness.

Theorem 1 Let HM be a homogeneous constant-rate hybrid machine satisfying Assumption 1 with initial state $x(0) = x_0 \in int(\mathbb{R}_+^m)$. Then HM is strongly Zeno if and only if $K\alpha \geq 0$ has no solutions in $\mathcal{A}$.

The condition of Theorem 1 (which is the standard feasibility condition for solution of a linear program) can easily be checked using standard available software. If $K\alpha \geq 0$ has solutions, the HM is not strongly Zeno and there exist switching policies resulting in non-Zeno runs of the system. However, without externally forced switching, the dynamic runs may still be Zeno. We discuss the control issues in the full version of the paper.

Although the problem of finding necessary and sufficient conditions for Zenoness (rather than strong Zenoness) is still open, we can solve the problem for *regular* systems, which satisfy both Assumption 1 and the following:

Assumption 2

The number of continuous (state) variables is equal to the number of configurations (that is, $n = m$). Each state variable is active in exactly one configuration. Furthermore, the rate matrix is of full rank (that is, $rank(K) = n$).

To present our results, let us consider all convex cones in $\mathbb{R}^n$ rooted at the origin. Denote by

$$\begin{aligned} & CONE(v_1, v_2, \ldots, v_l) \\ = \ & \{v \in \mathbb{R}^n : v = \beta_1 v_1 + \beta_2 v_2 + \ldots + \beta_l v_l \\ & \text{for some } \beta_1 \geq 0, \beta_2 \geq 0, \ldots, \beta_l \geq 0\} \end{aligned}$$

the convex cone generated by vectors $v_i \in \mathbb{R}^n, i = 1, 2, \ldots, l$.

Let $u_i = [0 \ldots 1 \ldots 0]^T$ be the n-vector with 1 in its ith position and 0 elsewhere. Denote

$$\begin{aligned} PO &= CONE(u_1, u_2, \ldots, u_n)(= \mathbb{R}_+^n) \\ NE &= CONE(-u_1, -u_2, \ldots, -u_n). \end{aligned}$$

If $rank[v_1 v_2 \ldots v_l] = r$, then the dimension of $CONE(v_1, v_2, \ldots, v_l)$ is r. Its boundary consists of r surfaces. Each surface is a part of a supporting hyperplane, generated by some $r - 1$ independent vectors in $\{v_1, v_2, \ldots, v_l\}$.

Lemma 2 Let C_1 and C_2 be two cones. If the surfaces of C_1 intersect C_2 only at the origin, then either C_2 is contained in C_1, or C_1 is contained in the complement of C_2.

Denote the column vectors of K by k_i: $K = [k_1 k_2 \ldots k_n]$.

Lemma 3 Under Assumption 2, the surfaces of $CONE(k_1, k_2, \ldots, k_n)$ and NE intersect only at the origin.

Lemma 4 Under Assumption 2, $K\alpha \geq 0$ has no solution in $\mathcal{A}$ if $K\alpha < 0$ has a solution in $\mathcal{A}$.

With these three lemmas, we can prove the following theorem that gives a necessary and sufficient condition for Zenoness of regular systems.

Theorem 2 Under Assumptions 1 and 2, a homogeneous constant-rate hybrid system HM is Zeno if and only if $K\alpha \geq 0$ has no solution in $\mathcal{A}$.

Note that for systems satisfying both Assumption 1 and Assumption 2, Zenoness and strong Zenoness are equivalent; that is, there exists a Zeno run of a system if and only if all its runs are Zeno. Also note that for systems satisfying Assumption 1 but not Assumption 2, no conclusion can be drawn just from the existence of solutions in $\mathcal{A}$ to the inequality $K\alpha \geq 0$, as to whether the system is Zeno or not.

Zeno behaviors have a complex nature even for systems satisfying Assumption 1 (but not Assumption 2) as we will illustrate by the following examples. Note that when the conditions of Theorem 1 or Theorem 2 are satisfied, then the results are independent of the initial conditions and the exact layout of connections between configurations. However, when these conditions are not satisfied, a dynamic run may or may not be Zeno depending on the initial conditions and on the exact layout of connections and guards between configurations. The first point is illustrated in Example 1.

Example 1 The system is shown in Figure 2.

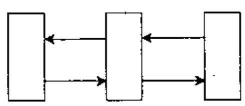


Figure 2: A system where Zenoness depends on the initial state

This system satisfies Assumption 1 but is not regular, since the second configuration has two active variables. Notice further, that while $K\alpha \geq 0$ has solutions in $\mathcal{A}$ and $K\alpha < 0$ has no solutions in $\mathcal{A}$, Zeno behaviors are possible. To understand the dynamic behavior of this system, observe that the loop consisting of configurations 1 and 2 (denoted by $1 \leftrightarrow 2$) has active variables x_2 and x_3. The submatrix corresponding to these variables is

$$K_{sub}^{L} = \begin{bmatrix} -90 & 130 \\ 1 & -90 \end{bmatrix},$$

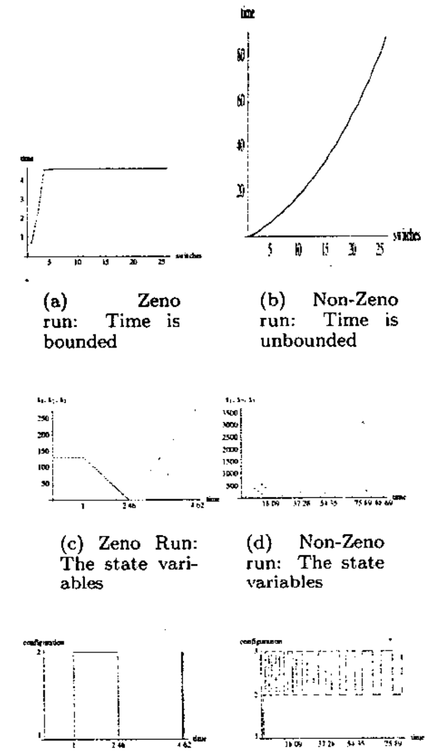
and represents a Zeno regular HM; that it, K_{sub}^{L} satisfies Assumption 2 and $K_{sub}^{L}\alpha \geq 0$ has no solutions in $\mathcal{A}_{sub}^{L} := \{\alpha_2, \alpha_3 | \alpha_2 \geq 0, \alpha_3 \geq 0, \alpha_2 + \alpha_3 = 1\}$. Thus, if a dynamic run is "trapped" in the loop $1 \leftrightarrow 2$, Zeno behavior must occur.

On the other hand, the loop $2 \leftrightarrow 3$ consisting of configurations 2 and 3, has active variables 1 and 2 with associated submatrix

$$K_{sub}^{R} = \begin{bmatrix} -90 & 70 \\ 130 & -90 \end{bmatrix}$$

which represents a non-Zeno regular HM ($K_{sub}^{R}\alpha \geq 0$ has solutions in $\mathcal{A}_{sub}^{R}$). Hence, if a dynamic run is "trapped" in the loop $2 \leftrightarrow 3$, it will be non-Zeno.

One can see that the system of Figure 2 will be trapped in one of the two loops after a number of initial transitions. Suppose that the initial configuration is 1. When $x_2 = 0$, a transition takes the system to configuration 2. Now suppose x_3 hits its guard before x_1 (i.e., $x_3 = 0$ is reached while $x_1 > 0$) and the system switches back to configuration 1, where the rate of x_1 is greater than the rate of x_3. After a while, the transition to configuration 2 takes place again, where x_1 and x_3 have the same negative rate, and therefore x_3 will again become zero before x_1, forcing the system back to configuration 1, and so on.



(a) Zeno run: Time is bounded

(b) Non-Zeno run: Time is unbounded

(c) Zeno Run: The state variables

(d) Non-Zeno run: The state variables

(e) Zeno run: Switching pattern

(f) Non-Zeno run: Switching pattern

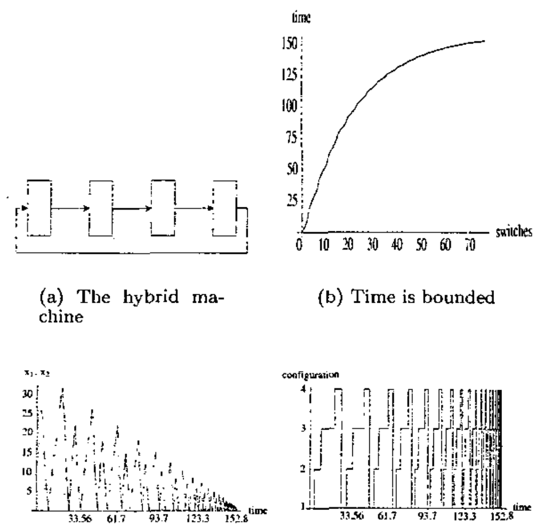
Figure 3: Representative Runs

(A) Zeno Run: $x_0 = [2, 90, 130]$, $q_0 = 1$; (B) Non-Zeno Run: $x_0 = [1, 90, 131]$, $q_0 = 1$

Thus, the behavior of the system is given by the matrix K_{sub}^{L}, corresponding to x_2 and x_3 in configurations 1 and 2. On the other hand, if after the first transition, x_1 becomes zero before x_3, a similar argument shows that the behavior depends only on the matrix K_{sub}^{R} corresponding to x_1 and x_2 in configurations 2 and 3. Therefore, we conclude that the run will or will not be Zeno, depending on the initial state. A simple calculation shows that, for $q_0 = 1$, the run is Zeno if $x_{0_1} > x_{0_3} - (129/90)x_{0_2}$, and it is non-Zeno if $x_{0_1} < x_{0_3} - (129/90)x_{0_2}$. In the case of equality, then after the first transition (from configuration 1 to configuration 2), both variables x_1 and x_3 become zero in configuration 2 at the same instant, and the system chooses its next configuration (either 1 or 3) nondeterministically, thereby becoming Zeno if it switches to configuration 1 and non-Zeno if it switches to configuration 3. Two sample runs that demonstrate Zeno and non-Zeno behaviors of this system are shown in Figure 3.

Example 2 This example shows that even for a Zeno system that has only one loop (and hence only one switching sequence), there may exist non-Zeno runs when switched properly.

The system is shown in Figure 4(a). Its dynamic run (i.e., when switched by the guards becoming true) is Zeno as shown in Figure 4(b)- Figure 4(d). However, $K\alpha \geq 0$ has solutions in $\mathcal{A}$. For example, one solution is $\alpha^* = [0.125, 0.125, 0.5, 0.25]^T$. Therefore, if the system is switched to remain in the proximity of the line emanating from x_0 in the direction of α^*, the run will be non-Zeno.



(a) The hybrid machine

(b) Time is bounded

(c) The state variables

(d) The switching

Figure 4: A Zeno system, for which $K\alpha \geq 0$ has solutions in $\mathcal{A}$

4 Conclusion

In this paper we studied various issues concerning the possible existence of Zeno behaviors in hybrid systems and the related question of existence of safety controllers that satisfy specified state invariance constraints. We focused our attention on constant rate hybrid machines, and showed that the existence of Zeno behaviors can be examined by checking for existence of solutions to a set of linear inequalities in a specified region of $\mathbb{R}^m$. In particular, we have shown that for the class of "regular" constant rate hybrid systems Zenoness is equivalent to strong Zenoness; that is, the system has Zeno runs if if and only if all its runs are Zeno. In this case, it is clear that if Zeno runs exist, no legal controller exists.

When a system has Zeno runs but is not strongly Zeno, some legal controller exists. However we have shown in the full version of the paper that, contrary to earlier belief, the existence of a legal (or safety) controller in Zeno systems does not always imply the existence of a minimally interventive (or minimally restrictive) controller. This implies, in particular, that the standard iterative synthesis algorithms that have been proposed in the literature may not apply in such cases. However, as was demonstrated in the full version of the paper, controllers can still be designed by more ad-hoc procedures.

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